Characteristics and hedonic pricing of differentiated beef demands

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Abstract

We estimate a CBS system model of U.S. derived demand for meat. We use this model (1) to examine relationships between three quality categories of beef and three other meats and (2) to simulate how taste shifts have affected demands for meats over time. We extend previous studies by disaggregating wholesale beef production into three quality categories: (1) USDA Choice grade or higher, (2) USDA Select or lower, and (3) cow and bull beef. Innovative features of our empirical model include a breakout of ‘table cuts’ into Choice and Select and the use of a hedonic characterization of the two breakouts to value “Choice-ness” and “beef-ness.” Our model demonstrates important shifts in separate demands for Choice and Select beef. We show that, separately, the demand for Choice-grade beef declined in the 1980s and 1990s and the demand for Select beef increased, a departure from the relatively stable demand characterizations of more aggregated measures of combined Choice and Select beef.

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1. Introduction

This article addresses two issues encountered in empirical work on demand for meat. First, empirical work is hampered by the absence of quality-differentiated quantity data at the retail level. Despite the fact that many commodities are graded and priced according to real or perceived quality differences, most demand studies aggregate commodities into undifferentiated, homogeneous products with single prices or otherwise fail to account for aggregate quality differences. Often aggregation across quality categories is necessary because data are relatively undifferentiated or are only incompletely available. A second issue is that shifts in demand may occur over time. Many people believe that U.S. red-meat demand started to decline sometime in the 1970s. Media reports starting in the late 1990s suggest that beef demand was increasing. The demand-shift terms used here allow for reversals in demand.

Regarding the first issue, a number of approaches are adopted in demand studies. In most studies addressing beef demand, beef is considered an undifferentiated commodity (Hahn, 1988; Alston and Chalfant, 1993; Brester and Wohlgenant, 1993; Eales and Unnevehr, 1993; Kesavan et al., 1993; Eales, 1994; Eales and Unnevehr, 1994; Hahn, 1994). In addition, a number of studies recognize that, in reality, beef quality varies greatly (Eales and Unnevehr, 1988; Heien and Pompelli, 1988; Brester and Wohlgenant, 1991; Capps et al., 1994; Huang and Hahn, 1995). Among these studies, some divide beef into basically two or more categories, especially hamburger and other cuts of beef (Huang, 1985; Eales and Unnevehr, 1988; Brester and Wohlgenant, 1991; and Huang and Hahn, 1995). However, their dependence on specific cuts restricts their usefulness to addressing only a small portion of aggregate beef demand.

In addition, at least three studies use micro-level or scanner data to look at demand for specific cuts of beef: Heien and Pompelli (1988) use survey data and Capps (1989) and Capps et al. (1994) use supermarket scanner data to analyze separate cuts and categories of beef. Unfortunately, these studies are concerned exclusively with beef purchased for home consumption, which differs in many respects from hotel, restaurant, and institutional consumption. Consequently, their results do not apply to the aggregate market.

Their studies and those of others have been restricted, in large part, because there are no adequate measures of aggregate retail quantities of beef separated by quality category. Noting the near one-to-one correspondence between wholesale and
retail quantities of beef, we use wholesale beef production, for which both aggregate price and quantity data by quality category are available, to overcome problems inherent in retail-level measures of quantity by quality.

An additional way to address quality issues in a seemingly homogeneous product is to examine the value of characteristics embodied in such a good (e.g., Atrostic, 1982; Bajic, 1993; Coatney, Mehkhaus, and Schmitz, 1996; Rudstrom, 2004; Bajari and Benkard, 2005; Chavas and Kim, 2005). In this framework, a good or commodity at time \( t \), \( x_t \), consists of a set of \( K \) characteristics, \( z_t = \{z_{1t}, z_{2t}, \ldots, z_{Kt}\} \), that are not directly priced in the market place, but which contribute to the total value of the good. The hedonic framework allows these \( z_{it} \) characteristics to be “priced.” We use a characteristics-based, or hedonic, specification for Choice and Select beef as a means of separately valuing the “Choice-ness” premium inherent in Choice beef and the generic “beef-ness” characteristic that exists in both Choice and Select beef.

Similarly, regarding the second issue, while numerous studies examine shifts in demand for various commodities and products (Tsurumi et al., 1986; Goodwin, 1992; Goodwin and Brester, 1995; Rickertsen, 1996; Moschini and Moro, 1996; Deschamps, 2003; Jouini and Boutahar, 2005), and despite the sophistication of the techniques employed, these studies generally consider only single shifts in demand. Moschini and Moro (1996) survey demand shift literature and describe a number of specifications used to model demand shifts. Deschamps (2003) uses time-varying intercepts to directly specify shifts in budget shares of consumer goods. Tsurumi et al. (1986), Goodwin (1992), Goodwin and Brester (1995), and Rickertsen (1995) use switching regression frameworks to specify shifts in a number of food and other commodities and goods.

More specific to our study, a number of studies examine the demand for beef in terms of responses to substitutes and other demand shifters (Choi and Sosin, 1990; Gao and Shonkwiler, 1993; Eales and Unnevehr, 1993; Sarmiento, 2005). Again, while these studies provide evidence of structural shifts in the demand for meats, they inadequately characterize these shifts because they generally address only single shifts in data series or adopt linear specifications for shifts. Others (Heinen and Pompelli, 1988; Capps, 1989; Capps et al., 1994) use data from relatively short time periods, which severely limits their ability to characterize longer-term shifts in tastes or consumption patterns.

We extend these and other earlier studies of meat demand by disaggregating wholesale beef into three quality categories, then use a theoretically consistent, derived-demand system (1) to analyze the demand for beef by quality and other factors and (2) to examine shifts in demand over the data period for each of three quality subcategories of beef, along with pork, chicken, and turkey. This model allows us to analyze the demand for beef by quality category and to demonstrate for each of the quality subcategories of beef and the other meats that multiple important shifts in demand are obscured by the use of more aggregate data. Features unique to this article include (1) the use of wholesale beef quantity data and prices by quality category, which represent aggregate beef disappearance and (2) the use of structural change parameters and variables that are designed to capture bidirectional shifts in demand.

2. Data

As pointed out above, a problem in implementing and estimating a differentiated beef-products model is finding appropriate price and quantity data. Aggregate retail beef quantities by quality category do not exist. We overcome this data deficiency by viewing retail quantities of beef as final products and wholesale-level quantities as inputs, where input quantities are reported by quality category. We divide U.S. wholesale beef into three broad quality categories: cutl (which roughly corresponds to the “hamburger” group of earlier studies and comes primarily from cows, bulls, and stags); “Choice” (which for our study includes both USDA Prime and Choice beef); and “Select” (which is the remaining steer and heifer beef). The basis for the usefulness of this approach is the near one-to-one correspondence between wholesale and retail carcass beef.

Wohlgenant (1989) and others provide additional support for this approach. Wohlgenant (1989) demonstrates that, under competitive conditions with constant returns to scale, derived demands meet the same restrictions as a consumer demand function. Wohlgenant and Haidacher (1989) lay out conditions under which derived demands can resemble retail-level demands: (1) if the processing sector is competitive and has constant returns to scale and (2) if marketing inputs are included in the demand system. Marketing inputs are necessary to address criticisms of specifications that assume proportional or combined proportional plus constant marketing margins in accounting for prices and inputs as products go from farm to retail (Wohlgenant, 1989; Wohlgenant and Haidacher, 1989). Wohlgenant and Haidacher (1989) also find that U.S. data are consistent with constant returns to scale in farm-to-retail food processing and marketing. Hahn and Green (2000) also find constant-returns-to-scale in wholesale-to-retail meat processing. Together, these findings allow us to use a consumer demand system, the CBS system, to estimate the wholesale-level, derived-demand for meat. In addition to the necessity of including a marketing input variable, our model further requires that consumer demand for meat be separable from the demands for other products, a hypothesis Moschini et al. (1994) test and do not reject.

2.1. Beef quality trends and consumer demand

To facilitate trade in beef, the United States Department of Agriculture (USDA) developed a beef-quality grading system that has been widely adopted by the beef industry and which the USDA has changed over time to meet changing needs. A change in 1976 allowed some lower grading (Good) beef to grade higher (Choice) (USDA, 1997). Few carcasses received
the Good grade, but instead were not graded (“no-rolled”\(^1\)), and it was extremely rare to see supermarkets sell USDA Good beef. The industry believed that consumers saw “good” products as “failed Choice.” In 1987, “Good” beef was renamed “Select” in an attempt to improve consumer acceptance, which appeared to have been a successful marketing strategy. It became common for packers to have Select carcasses graded, which made USDA Select beef more common in supermarkets.

At the same time, consumer health concerns increased the appeal of leaner meats, such as poultry and Select beef. This popularity of leaner meats persisted through the 1980s and 1990s. Since about 2000, the consumer view of marbling has again changed, along with a decline in the negative press regarding the healthfulness of red meat, with the result that the demand for highly marbled, Choice beef has increased. The Atkins’ diet and other popular high-protein diets are also given some credit for the recently observed increases in demand for higher-quality beef.

Only steers and heifers younger than about 42 months are eligible for the top USDA grades, Prime and Choice, and younger than about 30 months for Select (USDA, 1996). Most of the steers and heifers produced in the United States fall within the top three grades. Less than 5% of animals grade Prime, the highest grade (USDA, 2004). About half of the steers and heifers slaughtered in the United States grade Choice, the next highest grade (USDA, 2004). Most of the rest of the steers and heifers slaughtered in the United States grade Select, the third-highest USDA grade (USDA, 2004). Prime, Choice, and Select beef is sold through grocery stores as “table cuts.” The hotel, restaurant, and institutional (HRI) sector is the other important user of Prime-grade and the high-end Choice. Cows, bulls, and stags are another important source of beef and constitute most cattle and beef making up the remaining USDA grades, which we will call “cull” animals or “cull” beef. Cull-grade, or processing beef, ends up in hamburger and other manufactured beef products.

One of the most important factors distinguishing Prime, Choice, and Select-grade beef is the amount of marbling (intramuscular fat) in the meat. The more marbling the meat has, the higher its grade. Marbling requires that cattle be fed grain. If an animal has the genetics to make it into the higher grades, it requires more time on feed and more expense to achieve its potential. Since packers have no incentive to grade carcasses that are going to get “inferior” grades, Select values provide a floor for Choice prices. Packers could not grade Choice carcasses and sell them as no-rolls if the value of Select beef exceeded the value of Choice. Despite the increased demand in the 1990s for leaner beef, Choice cattle and beef maintain a price premium over Select. Part of this premium reflects the supply-side reality that Choice beef is more expensive to produce. Also supporting Choice demand in the 1980s was the growth of U.S. beef exports, especially to Japan, a market that demanded highly marbled beef. Another factor that has kept Select prices lower than Choice has been that cattle producers shifted to production practices, primarily a switch to genetically larger and leaner cattle, that expanded the supply of leaner cattle.

### 2.2. Disaggregated data

Sixty-eight observations of quarterly quantity and price data from first quarter 1988 through fourth quarter 2004 for Choice, Select, cull beef, pork, chicken, and turkey are used in the analysis. Of these variables, only pork and turkey prices and quantities and chicken quantities are stationary \(I(0)\) time series. The remaining variables are \(I(1)\) processes, stationary after first-differencing. Table 1 and Figs. 1 through 4 summarize characteristics of the data we use. Additional variables are created (below and Appendix 1) and used in the empirical analysis. We divide U.S. beef into three broad quality categories: Our “Choice” category is USDA Prime- and Choice-grade beef. The “Select” category is the remaining steer and heifer beef. About half of the steers and heifers slaughtered in the United States grade Choice or better (USDA, 2004). Most of the rest of the steers and heifers slaughtered in the United States are in the Select grade, the third-highest USDA grade (USDA, 2004). Not all cattle that can be graded Select are graded, and Stone (2004) finds that those that are not graded sell at the same price as Select. Cows, bulls, and stags are another important source of beef and constitute most cattle and beef making up the remaining USDA grades, which we designate “cull” animals or “cull” beef, and which ends up in hamburger and other manufactured beef products. This cull-grade beef roughly corresponds to the “hamburger” group of some earlier studies.

Our use of quarterly data introduces some potential for endogeneity between prices and quantities. To address this possibility, we estimate our model using instrumental variable techniques. Instruments include current and lagged personal consumption expenditures as reported by the Bureau of Economic Analysis, current and lagged consumer price indices (cpi) for all items, food, fish and sea-food, eggs, and dairy and related products. Other instruments are current and lagged soybean meal and corn prices reported by USDA, Agricultural Marketing Service (USDA, AMS). Of the instruments, only the cpi for eggs, personal expenditures, soybean meal prices, and population are stationary time series, although first differences of the other instruments are stationary.

Quantity data are USDA Economic Research Service (USDA 2005a,b) estimates of per-capita disappearance for pork, chicken, and turkey. We use USDA, AMS Grade reports (USDA, 2004) to split steer and heifer slaughter into “Choice” and “Select” grade beef. We assume that all imports are cull-grade beef and that all exports are Choice-grade beef. Per capita beef quantities are calculated by dividing each beef category quantity by population (USDA, 2005a). ERS gets mid-month population data from the Monthly National Population Estimates, Population Division, U.S. Census Bureau, U.S. Department of Commerce, as of April 19, 2004.

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\(^1\) “No-roll” refers to the fact that the grade stamp is on a roller which is run the length of the carcass.
Price data are also from the Red Meats Yearbook and Poultry Yearbook (USDA, 2005a,b) in which monthly prices, from USDA, AMS (2004) reports (unless otherwise noted), are averaged to obtain quarterly data. Prices include the price of Choice grade beef, for which we use the quarterly average of light Choice cuts reported monthly by USDA, AMS. Similarly, the Select-grade price is the light Select cutout value, and the cull beef price is the 90% lean trim price. Choice and Select beef quantities are measured on a boxed-beef basis. Cull beef quantity is measured as boneless beef. Pork price is wholesale pork composite price (USDA, 2005a). Chicken prices are the 12-city wholesale broiler-price series and turkey prices are the 3-area, wholesale turkey-price series from the Poultry Yearbook (USDA, 2005a,b), again compiled from USDA, AMS reports.

In order to fully implement Wohlgenant’s (1989) procedure for using a consumer demand model to estimate derived demands for meats consistent with Wohlgenant and Haidacher’s (1989) requirement that marketing inputs be included in this type of derived-demand system, we introduce measures of marketing costs. We create a matched set of marketing-cost indices for price and quantity as follows: We use USDA/ERS estimates of the wholesale-retail spreads for beef, pork, chicken, and turkey to estimate total expenditure on marketing inputs. We then scale the result by arbitrarily setting the first-quarter price of the marketing input to 1; the first quarter’s quantity of input is equal to its expenditure. We update the marketing-cost price by using a divisia price index based on the change in each species’ price spread weighted by each species’ share of total marketing input.

\[
\Delta \ln P_{im} = \sum_i \left( \frac{P S_i Q_{ij}}{\sum_j P S_j Q_{ij}} \right) \ln \left( \frac{P S_i}{P S_{i-1}} \right),
\]

(1)

where \( \Delta \ln P_{im} \) is the change in the logarithm of price index for the marketing input, \( P S_i \) is the price spread for meat \( i \) in quarter \( t \), and \( Q_{mi} \) is per-capita consumption of meat \( i \). The quantity of marketing input is, \( Q_{mi} \) is given by:

\[
Q_{mi} = \frac{\sum_j P S_{ij} Q_{ij}}{P_{im}}.
\]

(2)
2.2. Characteristics and hedonic pricing

As hedonic pricing is often used to explain how differentiated products are priced, based on their characteristics, we specify a model that, using a mix of characteristics’ demand and hedonic pricing, naturally incorporates the Choice-price-premium feature. Since cattle with the potential to be marketed as Prime and Choice can potentially produce Select beef by marketing them prematurely, that is, they possess the generic “beef-ness” characteristic, we want to know the value of feeding them to their potential grade. That is, we want to know the value of the “Choice-ness” characteristic.

We assume a set of two characteristics for Choice and Select beef and that the two products are priced using hedonic pricing. We then transform the data from its natural form to its hedonic form as follows: Since Choice is priced higher than Select,
we make Choice beef the equivalent of Select beef plus some enhancement. We call one the Generic characteristic and the other the Premium characteristic. Each pound of Select has a pound of Generic in it and no Premium. Each pound of Choice has a pound of Generic and a pound of Premium. We assign the difference between the price of Choice and Select as the value of Premium. The price of Generic is the price of Select. The hedonic pricing assumption implies that the Choice price is just the Generic price. Our quantity of Premium is the quantity of Choice and our quantity of Generic is the Choice quantity plus the Select quantity.

3. Beef demand modeling issues

We use the direct CBS (Central Bureau of Statistics) demand system specification developed by Keller and Van Driel (1985).
The CBS model is a differential model based on the derivatives of the demand system resulting from utility maximization, subject to a budget constraint. According to van Driel et al. (1997), “[t]he CBS model describes the differential change in the quantity share as a function of changes in real total expenditures and prices….” Barten and Bettendorf (1989) present direct and indirect versions of the CBS model and relate it to direct and indirect versions of the Rotterdam model and a differential version of the almost ideal demand system (AIDS). A full explanation of the CBS structure can be found in Keller and Van Driel (1985), and we present only a brief outline, below, specific to our problem.

The CBS has a number of advantages. It can be made globally consistent with the restrictions of demand theory, it is linear in its parameters, and it is a flexible functional form, that is, it can be made locally consistent with any set of demand quantities, prices, and elasticities. Finally, the CBS structure is the easiest framework for imposing negative semi-definiteness that assures proper signs on elasticities. Barten (1993) found the CBS specification to be preferable to the Rotterdam, Almost Ideal Demand, and NBER systems in several respects.

3.3. Empirical model structure

The CBS model is specified as a set of partial differential equations:

\[ w_i \left[ \partial \ln q_i - \sum_j w_j \partial \ln q_j \right] = \sum c_{ij} \partial \ln p_j + b_i \left[ \partial \ln x - \sum_j w_j \partial \ln p_j \right], \]  
(3)

\[ w_i = \frac{p_i q_i}{x}, \]  
(4)

where \( q_i \) is the quantity demanded of good \( i \), \( p_i \) is the price of good \( i \), and \( x \) the total expenditure on wholesale meat. The terms, \( \partial \ln q_i, \partial \ln p_i, \) and \( \partial \ln x \) are the derivatives of the logarithms of the quantity, price, and expenditure with respect to time. The \( w_i \) are expenditure shares. The \( b_i \) and \( c_{ij} \) are coefficients that, along with the meat’s share of total meat expenditure, determine the elasticities of demand. The expenditure elasticities depend on the \( b_i \) while the own and cross-price elasticities depend on the \( b_i \) and the \( c_{ij} \). In order to be consistent with demand theory, the coefficients must meet the following restrictions in accordance with Keller and Van Driel (1985):

\[ \sum_i c_{ij} = \sum_j c_{ij} = \sum_i b_i = 0, \]  
(5)

\[ c_{ij} = c_{ji}, \forall i, j. \]  
(6)

The restrictions in Eq. (5) make the model consistent with the budget constraint and homogeneous of degree 0 in prices and expenditures and Eq. (6) is the symmetry constraint which insures that the compensated demands are symmetric.

Demand theory implies that the matrix of compensated price derivatives is negative semi-definite (NSD) While the NSD restriction is not usually imposed in estimation of the CBS and related systems, one advantage of the CBS system over some differential systems (for example, the differential AIDS) is that the NSD restriction applies globally if the matrix of its price coefficients, the \( c_{ij} \), is NSD. In our model, we impose the restriction that the matrix formed by the \( c_{ij} \) must be NSD. Thus, we are assured, among other things that demands slope downward, that is, that compensated, own-price derivatives are negative, or at least not positive. The NSD restriction in our model is a nonlinear restriction. Because of the nonlinear constraints and inappropriateness of the OLS estimators, which are biased and inconsistent, we estimate our model using nonlinear three-stage least squares.

3.4. Taste shift variables and model structure

The system specified by equations (3) through (6) is a set of partial differential equations. However, since we observe prices and quantities, not derivatives, differential demand systems are estimated by assuming that differential systems are well approximated by difference systems. In equations (7) through (10), we use a straightforward first-difference form of our model to introduce our taste shifts variables and model specification before specifying the model we actually estimate (Eq. 12 below). Our taste shift variables allow shifts in two directions.

Differential models are often estimated with intercepts and other variables, including a number of transition functions designed to capture taste shifts and other structural changes into the intercepts of structural and vector autoregressive demand models (e.g., Tsurumi, Wago, and Ilmakunnas, 1986; Moschini and Meilke, 1989; Goodwin, 1992; Goodwin and Brester, 1995; Rickertsen, 1995; Moschini and Moro, 1996; Deschamps, 2003; Jouini and Boutahar, 2005; Holt and Craig, 2006). Most of the transition function specifications are capable of capturing only unidirectional shifts.

Taste shifts are easily incorporated into differential models. The model outlined in equations (3) through (6) contains only price and expenditure derivatives. Alston, Chalfant, and Piggott (2000) find that incorporating taste-shifts in the level version of the AIDS, while keeping it consistent with economic theory, requires one to make the price and expenditure coefficients functions of the advertising variables. They note that differential demand systems handle these types of variables more easily than demand systems specified in levels.

A nonzero intercept is interpreted as a trend in the demand for a product not driven by changes in price or income. These nonprice, nonincome changes are often attributed to taste shifts in differential models. For example, if the taste for meat \( i \) is a linear function of time,

\[ f_{it} + g_i, \]  
(7)
its derivative with respect to time is \( f_i \). Adding such a taste shift to Eq. (1) gives

\[
\begin{align*}
 w_i \left[ \partial \text{Ln}q_i - \sum_j w_j \partial \text{Ln}q_j \right] \\
= \sum c_{ij} \partial \text{Ln} p_j + b_i \left[ \partial \text{Ln} x - \sum_j w_j \partial \text{Ln} p_j \right] + f_i,
\end{align*}
\]

with the additional restriction,

\[
\sum_i f_i = 0, \tag{9}
\]

which assures that the adding up constraints hold. A positive \( f_i \) implies that demand for product \( i \) will increase over time, even if prices and expenditures are unchanged. This linear taste-shift term introduces an intercept to our CBS function.

At this point, it is useful to convert the differential equation in (8) into a difference equation relating demand in one period to the demand in the previous period (e.g., Theil, 1975, and van Driel et al., 1997),

\[
\begin{align*}
w_{it} & \left[ \Delta \text{Ln}q_{it} - \sum_j w_{jt} \Delta \text{Ln}q_j \right] \\
& = \sum c_{ij} \Delta \text{Ln} p_{jt} + b_i \left[ \Delta \text{Ln} x_{it} - \sum_j w_{jt} \Delta \text{Ln} p_{jt} \right] + f_i + e_{it}, \tag{10}
\end{align*}
\]

in which \( f_i \) is an intercept representing taste shifts. This conversion is necessary to move from the conceptual model to the empirical model because data consist of quantities and prices and differences, in our case, quarterly differences, are as close as one can get empirically to differentials in data.

Equation (10) includes an additive random error term, \( e_{it} \). This random error term can also be thought of as a random taste shift. It is an unmodeled, nonprice, nonexpenditure factor that randomly shifts demand. Error terms in this type of formulation induce a nonlinear type of unit root into the demand system. For example, if the error term increases demand by 5% in one quarter, that quarter’s ending demand is next quarter’s starting demand, and next quarter’s demand and each successive quarter’s demand is distorted upward. One way to address this problem is to switch from the first-difference form of the CBS to a “common-basis” (CBCBS) form, which requires replacing the intercept, \( f_i \), with a trend term. The trend term also makes the taste shifts monotonic. Introducing a cubic function of time extends the trend term in such a way as to address the monotonicity and also allows the taste-shifts to change direction twice. The trend term, thus, becomes \( \sum_k f_{ik} s_{it} \), where \( s_{it} \) is an annual weighting variable with \( k = 1, 2, \) or 3 for linear, squared, and cubic terms respectively (Appendix 1).

Strong seasonal patterns are evident in the quarterly U.S. data we used. For example, the demand for turkey exhibits a spike in the fourth quarter of the year. Such stable, long-term seasonal patterns of consumption are not generally taken as evidence of a shift in meat demand over time. We want to separate seasonal patterns from longer term shifts in seasonal patterns. We introduce three quarterly dummy variables, \( d_{it}, l = 1, 2, \) and 3, to capture seasonal behavior (Appendix 1). We dropped the first quarter’s dummy, and instead, in the first quarter of the year, we set the other three quarter’s dummy variables to \(-1\) so that the seasonal effects cancel themselves out over the course of a year. This assures that our seasonal specification allows seasonal changes to affect quarterly demand, but not annual demand. By specifying the seasonal terms this way, the coefficients for these dummy variables measure average seasonal patterns for the sample, which will allow us to extract average seasonality and examine variations from average seasonality.

Similar to the nonlinear treatment of the annual trend variables, we also want to test for shifts in the seasonality of demand. To accomplish this, we include interaction variables that are the dummy variables multiplied by trend and trend squared. These trend and trend-squared variables differ from the ones we use for annual-level taste shifts in that the seasonal shifters change every year rather than every quarter, although they also begin at \(-1\) and increase linearly to 1 (Appendix 1). We subtract the mean off the squared seasonal shifter so that it averages 0 over time and scale it so that it starts and ends at 1. All seasonal variables are represented in a new term, \( \sum_l g_{il}d_{il}, l = 1, 2, \) and 3, \( g_{il} \) are coefficients.

One of our goals is to measure the effects of taste-shifts over time on meat consumption, with a special emphasis on beef. Wholesale-level meat demand could be affected by both underlying consumer demand and the technology for processing wholesale meat into retail products. Separating out these two effects is difficult. However, the technologies for turning Choice and Select beef into consumer products are unlikely to be significantly different; therefore differences in our shift variables for Choice versus Select are more likely driven by consumer demand. We suspect that technology should, in general, lead to similar reductions in the costs of marketing all meat products. General improvements in meat-marketing technology would have the largest positive impact on the demand for those products with the highest marketing costs. Those meat products with more technical improvement imbedded in their marketing would gain relative to the others.

Equation (12) below is the form of the model we actually estimate:

\[
y_{it} = \sum_k (f_{ik} S_{it}) + \sum_l (g_{il} d_{il}) + \sum_j c_{ji} \text{Ln} \left( \frac{p_{jt}}{p_{jt}} \right)_{it} \\
+ b_i \left( \text{Ln} \left( \frac{x_{it}}{x_{ib}} \right) - \sum_j \left( \frac{1}{2} w_{jt} + \frac{1}{2} w_{jb} \right) \text{Ln} \left( \frac{p_{jt}}{p_{jb}} \right) \right) + e_{it}, \tag{12}
\]
where $s_{it}$ represents the three annual variables and $d_{ib}$ are the nine seasonal dummy variables, and $f_{ik}$ and $g_{ij}$ are their coefficients (Appendix 1). The subscript $t$ refers to a specific quarter and the subscript $b$ is the baseline value (see section 4.2 [Simulations] below for a description of the baseline). We use a common differential modeling convention of averaging current and baseline budget shares to create model variables. In (12), $y_{it}$ is output for the $i$th meat and is constructed as:

$$y_{it} = \left( \frac{1}{2} w_{it} + \frac{1}{2} w_{ib} \right) \times \left[ \ln \left( \frac{q_{it}}{q_{ib}} \right) - \sum_{j} \left( \left( \frac{1}{2} w_{jp} + \frac{1}{2} w_{jp} \right) \ln \left( \frac{q_{jt}}{q_{jb}} \right) \right) \right],$$

(13)

where variables are as previously defined.

Elasticities are derived by manipulating the appropriate derivatives from 12. The hedonic own- and cross-price elasticities of good $i$ with respect to price $j$ are, thus, calculated as follows (van Driel et al., 1997):

$$\varepsilon_{ij} = \frac{c_{ij}}{\bar{w}_{ij}} - \varepsilon_{ix} \bar{w}_{j},$$

(14)

and the expenditure elasticities are similarly calculated as:

$$\varepsilon_{ix} = \frac{b_{x}}{\bar{w}_{ix}} + 1$$

(15)

where $\varepsilon_{ij}$s are the own- and cross-price elasticities, $\varepsilon_{ix}$s are expenditure elasticities,

$$\bar{w}_{ix} = \frac{1}{T} \sum_{t} \left( \frac{1}{2} w_{it} + \frac{1}{2} w_{ib} \right),$$

and all other variables and all parameters are as previously defined.

4. Results and discussion

This section describes results based on the use of variants of the model in (12) to estimate parameters (Table 2) that we use in the following capacities: First, the remainder of this section discusses the basic demand structure for beef and other meats in the form of demand elasticities calculated from parameters estimated with the model in (12). Second, several restrictions, outlined below, allow us to test several variants of demand stability. Third, we use the set of restrictions that, together, form the “best” model, which forms the basis for simulations that allow us to determine how demands have shifted over the study period.

4.1. Elasticity estimates

The basic structural information extracted from model results consists of the own- and cross-price elasticities and expenditure elasticities. Hedonic elasticities for beef (Premium and Generic) are presented in Table 3 and more traditional forms of elasticities are presented in Table 4 (see Appendix 2 for transformation details). Elasticities in Tables 3 and 4 are ordinary demand elasticities. Demand and expenditure elasticities for the hedonic model are based on the estimated CBS parameters and average budget shares for the sample period with symmetry, homogeneity, and the budget constraints imposed (Table 2). Symmetry in the cross-price coefficients does not translate into symmetric cross-price elasticities, nor do they have to have the same sign. Many elasticities are close to zero, and most (hedonic) elasticities are not significant (Table 3), despite a number of significant parameter estimates (33% of parameters at the 5% level of significance and 40% at the 10% level, Table 2).

There are at least two reasons why statistically significant parameter estimates might result in insignificant elasticity estimates: The elasticities are calculated from parameter estimates, in particular, the expenditure elasticities, none of which are significant (except the expenditure parameter for Premium, which is significant at the 10% level), and their associated weights (see equations (14) and (15)). Further, variances for the parameters associated with meat expenditures are also relatively large. Own- and cross-price elasticities and their variances incorporate expenditure elasticities and their associated variances and weights in ways that could result in fewer elasticities being significant than parameter estimates would suggest at first glance.

Second, in many applied studies, variances of parameter estimates are based on asymptotic distribution theory. The Rao approach is one such asymptotic approximation. Green et al. (1987) show that the bootstrapped covariances have better small-sample properties. The elasticities for CBS are not nonlinear functions of the coefficients; our bootstrapping technique differs from the asymptotic approach only to the extent that our bootstrapped coefficient covariance matrix differs from the NL-3SLS covariance matrix. Since the NL-3SLS covariance matrix itself has only asymptotic properties, the small-sample distributions are likely to differ from the asymptotic distributions, which raise questions about results using the variance-covariance matrix from the NL3SLS runs. Because of the uncertainty surrounding the model variance-covariance estimates, we use bootstrapping to give us small-sample distributions with better-known distributional characteristics. Other ways of addressing statistical significance issues we did not explore include increasing sample size and alternative model specifications.

Finally, as we discuss below, our elasticities match closely with elasticities from the literature for meat cuts that most closely compare with our meat categories. Because of this consistency in estimates between our results and those of other studies and despite the lack of significance, we think our results are useful.

All own-price elasticities have the expected negative signs, and all own-price hedonic demand elasticities are inelastic. The most elastic is $-0.484$ for generic beef. The least elastic is $-0.041$ for Premium. Our own-price elasticity for Premium...
CBS taste-shift, seasonal, and parameter estimates for U.S. meat demand, 1988–2004

<table>
<thead>
<tr>
<th>Annual taste shifters</th>
<th>Premium</th>
<th>Generic</th>
<th>Cull</th>
<th>Pork</th>
<th>Chicken</th>
<th>Turkey</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend</td>
<td>−0.0001</td>
<td>−0.0018</td>
<td>0.0002</td>
<td>0.0011</td>
<td>0.0001</td>
<td>−0.0002</td>
<td>0.0007</td>
</tr>
<tr>
<td>Squared trend</td>
<td>−0.0002</td>
<td>0.0007</td>
<td>−0.0026</td>
<td>−0.0016</td>
<td>0.0003</td>
<td>0.0003</td>
<td>−0.0041</td>
</tr>
<tr>
<td>Cubed trend</td>
<td>0.0000</td>
<td>0.0001</td>
<td>−0.0001</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Seasonal effects

| Quarter 2 | 0.0003 | 0.0004 | 0.0005 | 0.0006 | 0.0004 | 0.0003 | 0.0002 |
| Quarter 3 | 0.0000 | 0.0000 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Quarter 4 | −0.0005 | −0.0010 | −0.0008 | 0.0056 | 0.0014 | 0.0007 | −0.0032 |
| Times trend | −0.0001 | −0.0007 | 0.0004 | 0.0003 | 0.0002 | 0.0000 | 0.0001 |
| Times square trend | 0.0000 | 0.0020 | 0.0005 | 0.0014 | 0.0014 | 0.0007 | −0.0032 |

Seasonal effects

Taste-shift and seasonal estimates of t-ratios based on 200 bootstrap iterations

| Premium | −0.89 | −2.02 | 0.96 | 1.96 | −1.98 | −6.00 | 6.20 |
| Generic | −2.02 | −5.06 | 1.67 | 1.78 | −8.14 | −0.77 | 0.91 |
| Cull    | 0.44  | 3.59  | 2.19 | −1.40 | 2.80  | −3.19 | 0.77  |
| Pork    | 1.92  | 1.65  | 1.51 | −0.99 | −1.27 | −0.25 | −1.37 |


T-ratios, based on 200 bootstrap iterations

(−0.041) is small. Our elasticity for Generic beef is lower than Capps’ (1989) estimate for Roast beef (−1.27) and Brester and Wohlgemant’s (1991) estimate for Other beef (−0.81), which, of the two, is more similar to our Generic beef variable. Our elasticity for Cull-grade beef (−0.148) compares favorably with Capps’ (1989) estimate for Ground beef (−0.15), but is more inelastic than most other estimates: Huang and Hahn’s (1995) elasticity for Manufacturing grade beef (−0.401), Eales and Unnevehr’s (1988) estimate for Hamburger (−2.59), Heien and Pompelli’s (1988) estimate for Ground beef (−0.85), and Brester and Wohlgemant’s (1991) estimate for Ground beef (−1.015). Both Premium and Cull are relatively more inelastic than Generic beef, which means that both high quality and low quality attributes are less responsive to price changes than generic beef. For Premium beef, this makes sense, especially combined with its high expenditure elasticity, indicating it is a luxury good, and price may not matter (up to a point).

Own- and cross-price elasticities for Generic, and Cull are progressively more inelastic (column 1, Table 3), and a roughly similar progression toward the inelastic range occurs with respect to the price of marketing inputs (column 7, Table 3). These progressions are consistent with the notions that quality substitutes for quantity as raw product prices rise and reduced output of the commodities using less of a relatively higher-priced input (Barzel, 1976). These patterns are even more obvious in Table 4 and include Choice beef, perhaps because Choice beef accounts for 7% of the meat market.
for both the Premium characteristic and half the Generic characteristic.

All the expenditure elasticities in Table 3 are positive, and expenditure elasticities for Premium, Generic, and Chicken are greater than 1, implying that these meats are luxuries (e.g., Rickertsen, 1996). The expenditure elasticity for Premium (Table 3) and Choice beef (Table 4) is more than twice the magnitude of the next highest estimate, and suggests that Choice-ness is viewed as more of a luxury than Generic or Cull-grade meat.

Because Cull-grade beef, Pork, Chicken, Turkey, and the Marketing input are the same in both hedonic and traditional forms, the transformation does not affect the own- and cross-price elasticities for these products. Compensated demand for Choice beef is relatively price elastic, with relatively large cross-price elasticities—products with good substitutes tend to have relatively elastic demands. Our traditional demand elasticities for Choice and Select are smaller than Huang and Hahn’s (1995) elasticity for High-quality beef (−1.036), which actually consists of both our Choice and Select beef. Our estimates compare most favorably with Heien and Pompelli’s (1988) estimate for Steak (−0.73) and Capps’ (1989) elasticity for Steak (−0.72).

Our cross elasticities for Generic beef and prices for pork and chicken compare favorably with Eales and Unnevehr (1988), Eales and Unnevehr (1993) (Pork), Gao and Shonkwiler (1993), Kesavan et al., (1993), Hahn (1994), but are lower than those reported by Huang and Hahn (1995). Signs on estimates from Hahn (1988), Brester and Wohlgenant (1991) (pork), Brester and Wohlgenant (1993), Eales and Unnevehr (1993) (chicken) as well as Moschini, Moro, and Green (1994) are positive, while ours are negative. We interpret this to mean Generic beef is not a good substitute for pork or chicken. We report positive cross elasticities for Generic beef quantities and turkey prices and for Cull beef quantities and Premium beef, Generic beef and pork prices, and for pork quantities and Premium prices, implying that these meats substitute for one another at some level.

4.2. Stability of demand

In addition to the demands for meats, we are interested in testing hypotheses of the stability of demand for one or more of the products. Results shown in Tables 2 and 5 demonstrate...
significant shifts in meat demands for all meats except pork. These results are consistent with Moschini and Meilke (1989), Choi and Sosin (1990), Gao and Shonkwiler (1993), and Rickertsen (1996), although others have found no evidence for structural shifts (Eales and Unnevehr, 1993; Huang and Hahn, 1995).

To further characterize stability in meat demand, we estimate an unrestricted model (model 1 (Table 2), not shown in Table 5) and three restricted models (Table 5, models 2, 3, and 4) for each commodity, and conduct Chi-square tests on differences in weighted sums of squares between pairs of models. Each model consists of the entire system of equations, and the entire system of equations is estimated for each set of restrictions on each meat. The $f_{it}$ coefficients determine whether or not taste shifts cause changes in annual demand. The $g_{it}$ coefficients associated with the trend and trend-squared terms measure how demand seasonality changes over the sample period. In all, we estimate 22 different models, testing each for stability.

Model 1 is the most general model, imposing no restrictions on $f_{it}$ or $g_{it}$ (Table 2). The other 21 models are the result of applying one of three sets of restrictions on each meat. Comparing results from each of the 21 restricted models with model 1 enabled Chi-square tests for different levels of stability (Table 5). For a meat’s demand to be totally stable, all three $f_{it}$ coefficients and the six $g_{it}$ coefficients associated with the trend and trend squared terms are set to 0 (Model 2). For Model 3 results, we only require that the meats have a stable seasonal pattern (the $g_{it}$ coefficients are set to 0), but allow annual demand to shift. In Model 4, the annual demand is stable (the $f_{it}$ coefficients are set to 0), but the seasonal pattern is allowed to shift. Model 2 is the most restricted set of models and is nested in both Models 3 and 4.

In Table 5, failure to reject the alternative hypothesis means that the stability restriction appears to be valid. Most of the stability tests are rejected. None of the products appear to be totally stable over the sample period. The only nonsignificant tests in Table 5 are the “stable seasonal” (Model 3) restrictions for Premium, Cull-grade beef, and Chicken.

4.3. Simulations

While results for all six meats and the marketing input demonstrate statistically significant shifts in tastes, specific patterns of taste shifts over time are also of interest. To analyze these patterns over time, we simulate taste-shift results using the unrestricted model coefficients described in the previous section as Model 1. Because prices and expenditures vary along with tastes, we fix prices and expenditures at baseline values and simulate the effects of taste shifts. We also extract average seasonal patterns, leaving simulations that emphasize shifts, including seasonal shifts, that are departures from average seasonality.

The original development of differential models begins with price and expenditure derivatives that need not be functions of time. When moving from the differential to the difference form, analysts select the first-difference format as a matter of convenience, the previous period’s value serving as a baseline for the next period. Mathematically, the choice of a set of baseline values for comparing prices and quantities over time is arbitrary. A common “baseline” provides a means of simultaneously comparing all prices and quantities across time.
For our baseline, we select the geometric mean of prices and quantities (and meat expenditures implied by these prices and quantities). We also use a fourth-order auto-regressive specification for the error terms; the lag structure is common across all equations. Our estimated first- and second-order autoregressive terms are not statistically significant; the third- and fourth-order coefficients are. We measure standard errors by bootstrapping the model using all four of the estimated lag coefficients. The nonsignificance of the first two autoregressive coefficients supports our use of the common-baseline CBS. Differencing is often used to eliminate unit roots. If a difference-type model were to be appropriate here, our lack of differencing should have given us error vectors with large and significant first-order autoregression.

Our Choice and Select data are on a boxed-beef basis, while cull-grade beef are measured on a boneless-weight basis. These are transformed to a common, retail-weight basis using USDA conversion factors and presented in Fig. 5. Fig. 6 shows the effects of taste shifts for the other meats.

Fig. 5 provides a clear picture of why it is important to disaggregate beef quantities in demand studies. In Fig. 5, total beef consumption over the period declines steadily from 19 pounds per quarter at the start to around 16.5 pounds per quarter at the end of the period. Demand for cull-grade beef declines smoothly by half a pound over the same period. Most of the change is due to declines in the demands for Choice and Select beef. Choice beef starts out a higher level relative to Select; the two were roughly equal for the mid-point of the sample. Select beef demand exceeds Choice from 1997 to 2002, when Choice demand recovers somewhat.

Choice and Select beef also show changes in seasonality that are not as evident in total beef demand. Most of the seasonal variation in Choice demand offsets seasonal variation in Select demand—the estimated shift in the seasonality for generic has a small effect on the demand for generic beef (Choice plus Select). Generic beef’s seasonal shift is statistically significant despite its small actual effect. On the other hand, the change in seasonality for Premium beef is not statistically significant but has a noticeable effect on how demand is divided between Choice and Select beef.

Pork and chicken show the clearest shifts in consumption (Fig. 6). Pork demand declines by 1.5 pounds through out the sample period while chicken demand increases by almost 8 pounds. Pork and beef are more expensive meats than chicken at wholesale, so shifting expenditures from beef and pork to chicken increases pounds of chicken consumption by more than it decreases red meat consumption. Turkey shows minor changes in yearly consumption. Toward the end of the sample, the seasonal declines in tastes occur in the fourth quarter. Not unexpectedly, turkey demand is still strongest in the fourth quarter of the year, although the apparent seasonal declines reflect the fact that turkey consumption is more evenly distributed over the course of a year.

The patterns for Choice beef, Select beef, chicken, and pork are consistent with prior expectations in which human health concerns motivate a reduction in demand for Choice beef and pork relative to leaner meats such as Select beef and chicken. Despite demonstrating some evidence of a shift back toward Choice beef near the end of the sample, perhaps in response to the popularity of high-protein diets and relaxed concerns about
the human health impacts of fatty, red meat, tastes for Choice beef at the end of the sample do not recover to their peak values of the early 1990s.

5. Conclusions

The primary goal of this study is to test for taste shifts for beef, focusing on the demand for beef by quality category. We do this in two ways: First, our modeling framework estimates changes in meat demand quantities, given price and expenditure changes, and in which beef is disaggregated into three quality classes, or more accurately, using demand for characteristics. The hedonic framework captures the Choice beef price premiums that are invariably higher than Select, as well as the fact that both Choice and Select have a common “beef-ness” characteristic. Second, we simulate our data series over time, extracting average annual and seasonal shifts, in order to examine demand shifts over time.

Comparing results for different levels of aggregation of the beef categories demonstrates the value of the disaggregated view of beef as more than one commodity. We conclude that total beef demand has been relatively stable since the 1980s except for some small changes in seasonality. However, disaggregated beef consumption data demonstrate a significant departure from aggregate stability. Beginning in the 1980s, disaggregated beef demand shifts away from Choice and into Select beef. Late in the data series, this trend appears to reverse, and Choice beef regains some of the demand it had lost relative to Select. We surmise that supply responses in the form of genetic shifts in cattle contributed to these demand shifts during the 1980s and early 1990s and the shift back to increased marbling in the mid-to-late 1990s (Grant, 1998). Other meats follow trends consistent with this human health, fat-negative story of demand shifts: Pork demand declines over the time period, and chicken demand increases.

Endogeneity persists in our specification as quantities of meat are endogenous in our specification in which all meats appear in each equation. Endogeneity makes least-squares type estimates subject to bias and inconsistency. For this reason and because some of the parametric constraints are nonlinear, we estimate the demand model using nonlinear, three-stage least squares.

One limitation of our study is that our taste shifts are a combination of consumer taste shifts as well as shifts in the wholesale-to-consumer meat marketing technology. However, the total shifts observed are consistent with prior expectations about consumer demand shifts. In addition, we find shifts in seasonality in the demand for meats, and it is hard to see how meat-processing technology would have a strong seasonal component.

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Appendix 1

We set our common baseline taste to 0 for each meat by arbitrarily starting our linear trend variable at −1 and increase it to 1 by the end of the sample period, wherein both the linear trend and its cube average 0 for the sample period. In order for the squared term to average 0, we subtract the mean of the squared term from each period’s squared value, then rescale it so that the quadratic trend starts and ends at 1:
where \(s_{lt} \) represents three annual variables, \(k = 1, 2, \) or 3 (linear, squared, and cubic terms), and are defined as follows:

\[
\begin{align*}
    s_1 & = s'_1 = 1,000, \\
    s_t & = s'_t = s'_{t-1} + 2/(T - 1), \\
    s^2_t & = \frac{s^2_t - 1}{T} \sum_{t'} s^2_{t'}, \\
    s^3_t & = (s'_t)^3.
\end{align*}
\]

and scale it so that it starts and ends at 1. All seasonal variables are represented in a new term, \(\sum_t g_{lt} d_{lt} \), 1 = 1, 2, and 3, \(g_{lt}\) are coefficients, and matrices \([a_{il}]\) and \([z_{il}]\) are combined in such a way as to produce a 9 \(\times 68\)-dimensioned matrix, \([d_{il}]\), of seasonal variables:

\[
[a_{il}] = \begin{bmatrix}
-1 & -1 & -1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix},
\]

\[
[z_{il}] = \begin{bmatrix}
1 & -1.000 & 1.000 \\
1 & -0.875 & 0.625 \\
\vdots & \vdots & \vdots \\
1 & 1.000 & 1.000 \\
\end{bmatrix}_{3 \times 17},
\]

where

\[
z_{2t} = z'_{2t-1} + 4/(T - 4), \text{ for } t > 4, \text{ and}
\]

\[
[z'_{21}, z'_{22}, z'_{23}, z'_{24}]^T = [0 \ 0 \ 0]^T.
\]

Similarly,

\[
z_{3t} = z^2_{3t-1} + 4/(T - 4), \text{ for } t > 4,
\]

\[
[z^2_{21}, z^2_{22}, z^2_{23}, z^2_{24}]^T = [0 \ 0 \ 0]^T.
\]

Note that the denominators in both \(z_{2t}\) and \(z_{3t}\) are the first observation minus a mean. The direct, or Kronecker, product of \([z_{il}]\) and \([a_{il}]\) yields \([d_{il}]\):

\[
[d_{il}]_{9 \times 68} = [Z_{il}] \times [a_{il}] =
\]

\[
\begin{bmatrix}
-1.00 & 1.000 & -1.000 \\
1.000 & 0 & 0 \\
0 & 1.000 & 0 \\
0 & 0 & 1.000 \\
-1.000 & 0.875 & -0.625 \\
\vdots & \vdots & \vdots \\
0 & 0 & 1.000 \\
\end{bmatrix}
\]

The first three columns represent the seasonal dummy variables, the third mid columns represent the dummy by trend interaction variables, and the last three columns represent the dummy by trend squared interaction variables.

**Appendix 2**

The weighted inversion of hedonic elasticities to traditional elasticities is facilitated by the fact that the elasticity submatrices for cull beef, pork, chicken, and turkey are the same under both regimes. The procedure requires stacking three matrices.
and inverting. To see this, envision the following system consisting of 21 equations (each line below is a 7-equation system):

\[
\begin{align*}
\partial Lpq^h &= \varepsilon_{ij} \partial Lnp^h + \varepsilon_{hx} \partial \ln x, \quad \text{(A.1)} \\
\partial Lpq^h &= M_1 \partial Lnp^h, \quad \text{(A.2)} \\
\partial Lnp^h &= M_2 \partial Lnp^r, \quad \text{(A.3)}
\end{align*}
\]

where \( q^h \) is a vector of hedonic quantity, \( p^h \) is a hedonic price vector, \( q^r \) is a “regular” quantity vector, \( p^r \) is a “regular” price vector, \( \varepsilon_{ij} \) is a price elasticity matrix, \( \varepsilon_{hx} \) is the expenditure elasticity matrix, and \( M_1 \) and \( M_2 \) are modified identity matrices with quantity and price weights, respectively, for Choice (and Prime) and Select replacing the ones in the first two diagonal cells. By appropriately stacking the above system and rearranging, we get:

\[
\begin{bmatrix}
I & -\varepsilon_{ij} & 0 \\
I & 0 & -M_1 \\
0 & I & 0
\end{bmatrix}
\begin{bmatrix}
\partial Lpq^h \\
\partial Lnp^h \\
\partial Lnp^r
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
M_2
\end{bmatrix}
\begin{bmatrix}
\partial \ln p^r \\
\partial \ln x
\end{bmatrix}
\]

\begin{align*}
\partial Lpq^h &= \partial Lnp^h + \varepsilon_{hx} \partial \ln x. \quad \text{(A4)}
\end{align*}

Letting

\[
A = \begin{bmatrix}
I & -\varepsilon_{ij} & 0 \\
I & 0 & -M_1 \\
0 & I & 0
\end{bmatrix}
\quad \text{and} \quad
B = \begin{bmatrix}
0 \\
0 \\
M_2
\end{bmatrix}
\]

\[
\begin{align*}
\partial Lpq^h &= A^{-1} B \partial Lnp^r + A^{-1} \left[ \begin{array}{c}
\varepsilon_{hx} \\
0 \\
0
\end{array} \right] \partial \ln x, \quad \text{(A5)}
\end{align*}
\]

in which the elements of the partition pertaining to \( \partial Lpq^h \) and \( \partial Lnp^r \) on the right-hand side are the elasticities of interest.

References


