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Developing an Empirical Yield-Prediction Model Based on Wheat and Wild Oat (*Avena fatua*) Density, Nitrogen and Herbicide Rate, and Growing-Season Precipitation

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To develop a more complete understanding of the ecological factors that regulate crop productivity, we tested the relative predictive power of yield models driven by five predictor variables: wheat and wild oat density, nitrogen and herbicide rate, and growing-season precipitation. Existing data sets were collected and used in a meta-analysis of the ability of at least two predictor variables to explain variations in wheat yield. Yield responses were asymptotic with increasing crop and weed density; however, asymptotic trends were lacking as herbicide and fertilizer levels were increased. Based on the independent field data, the three best-fitting models (in order) from the candidate set of models were a multiple regression equation that included all five predictor variables ($R^2 = 0.71$), a double-hyperbolic equation including three input predictor variables ($R^2 = 0.63$), and a nonlinear model including all five predictor variables ($R^2 = 0.56$). The double-hyperbolic, three-predictor model, which did not include herbicide and fertilizer influence on yield, performed slightly better than the five-variable nonlinear model including these predictors, illustrating the large amount of variation in wheat yield and the lack of concrete knowledge upon which farmers base their fertilizer and herbicide management decisions, especially when weed infestation causes competition for limited nitrogen and water. It was difficult to elucidate the ecological first principles in the noisy field data and to build effective models based on disjointed data sets, where none of the studies measured all five variables. To address this disparity, we conducted a five-variable full-factorial greenhouse experiment. Based on our five-variable greenhouse experiment, the best-fitting model was a new nonlinear equation including all five predictor variables and was shown to fit the greenhouse data better than four previously developed agronomic models with an R^2 of 0.66. Development of this mathematical model, through model selection and parameterization with field and greenhouse data, represents the initial step in building a decision support system for site-specific and variable-rate management of herbicide, fertilizer, and crop seeding rate that considers varying levels of available water and weed infestation.

Nomenclature: Imazamethabenz; wild oat, *Avena fatua* L. AVEFA; wheat, *Triticum aestivum* L.
Key words: Yield prediction, empirical modeling, site-specific management, fertilizer, herbicide, precipitation, precision agriculture.

Management decisions made by many farmers are still based mainly on tradition, personal observations, and interaction with crop consultants, industry salespersons, and university extension personnel (Anderson 2003). Precision agriculture technologies may provide a means to integrate farmer knowledge with site-specific tools. Site-specific management may, in turn, increase efficiency of resource use and optimize net returns. On-farm and on-research station experiments have suggested improved economic gain with spatially targeted, variable application rates of fertilizer (Barton 1992; Li and Yost 2000) and herbicide (Grundy et al. 1996; Johnson et al. 1995; Walker et al. 2002). In addition, environmental pollution and selection for herbicide resistance can be reduced (Christensen et al. 1998; Jasieniuk et al. 1999) if variable-rate applications of fertilizer and herbicide are made only when previous site histories (e.g., if a specific location in the field is known to be nitrogen rich or poor) and threshold weed densities warrant their use.

Mechanical tools and information sensors for site-specific management (e.g., yield monitors, variable rate sensors, etc.) are under development, but there is a critical need to further understand the underlying ecological processes involved in optimizing grain yield. A wheat-yield model, based on the mechanisms of plant competition for limited resources in a variable environment, has not previously been developed. With deeper knowledge of the factors influencing yield, more accurate herbicide and fertilizer prescriptions could be made.

Our research attempted to elucidate ecological first principles, including each variable's influence on yield and the interactions between independent variables, such that precision agriculture technologies can be employed to optimize site-specific management of farm inputs. The first principles of a science are generalizations based on many empirical observations, that is, that a particular, consistent outcome (first principle) has a logical explanation. Thus, first principles become the basic tenants that form the foundation of a science discipline. We have focused on five highly influential and easily measured predictors of wheat yield: wheat density, wild oat density, nitrogen rate, herbicide rate, and available water. An effective yield-prediction model would serve as the core of a decision support system, through which, site-specific management strategies of nitrogen, fertilizer, and crop seeding rate could be recommended.

Historically, studies addressing the generality and predictive power of relationships have not been thoroughly explored in agronomy or agroecology (Beck 1997). Some yield-prediction modeling in agronomy has made significant progress in quantifying crop–weed interactions. Early yield models, resembling simple linear-regression equations that included weed density, were developed by Bleasdale and Nelder (1960), Holliday (1960), and Farazdaghi and Harris (1968), among many others (see reviews by Willey and Heath [1969], Cousens [1985a, 1985b], and Firbank and Watkinson [1990]). In recent decades, more commonly used models include the yield–weed density and the yield–loss models described by Cousens (1985a, 1985b). An alternative strategy

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for yield models pioneered by Shinozaki and Kira (1956) included crop density as well as weed density. Later, Firbank and Watkinson (1985) developed a two-species competition model, and Maxwell and Jasieniuk (Jasieniuk et al. 2000) developed a double-hyperbolic, yield-prediction equation also expressing two-species competition.

Several studies have included the population dynamics of weed impacts on crops to capture the multiple-year effects of the weeds (Cousens et al. 1986; Gonzalez-Andujar and Perry 1995; Mortimer 1987; Wilson et al. 1984). Other researchers have considered how well historical weather data by themselves can predict wheat yield (Brooks et al. 2001; Chipanshi et al. 1997; Hammer et al. 2001; Haun 1974; Williams 1973). Until recently, no empirical, mechanistic, or population-dynamics model has included the combined effects and interactions of uncontrolled environmental resources and controlled agricultural inputs on yield.

Brain et al. (1999) and Kim et al. (2002) reported models that include herbicide application with weed density as predictor variables of crop yield. Another important advancement in yield modeling was developed by Kim et al. (2006), who modeled the effects of sublethal doses of herbicide and fertilizer on crop–weed competition. Although yield model development has progressed in weed science, the environmental variable of water, arguably the most influential variable on yield, has not been included in crop–weed competition models that also include fertilizer and herbicide. The only yield models that have included agricultural inputs, such as fertilizer, and environmental variables, such as weather, are crop-growth models such as the Crop Estimation through Resource and Environment Synthesis (CERES), developed by Ritchie and Otter (1985) and Godwin et al. (1990). Later Beckie et al. (1994), Chipanshi et al. (1997), and Moore and Tyndale-Biscoe (1999) used CERES to investigate wheat growth over a range of weather conditions, fertilizer rates, and soil types. Beckie et al. (1994) also tested the effectiveness of three other simulation models (e.g., the Erosion Productivity Impact Calculator [EPIC], the Nitrate Leaching and Economic Analysis Package [NLEAP], and Nitrogen–Tillage–Residue-Management [NTRM]) for estimating nitrates and water in two soils. Although mechanistic models, such as CERES, EPIC, NLEAP, NTRM, and the crop–weed INTERspecific COMpetition model, INTERCOM (Kropff and van Laar 1993) among others, are extremely valuable to the investigation of physiological and phenological processes, they are generally not as suitable as empirical models for management in agriculture because they require estimating more parameters without yet showing better predictions than empirical models (Barnett et al. 1997).

As with modeling exercises, experimental studies have typically included only a small number of predictors. Many field studies in agronomy have investigated how available nitrogen (Henry et al. 1986; Racz 1974), available water (Bauder et al. 1987; Brown and Carlson 1990; DeJong and Rennie 1967; Lehane and Staple 1965), or herbicide (Salonen 1992; Spandl et al. 1997) individually influence wheat yield. Many other studies have explored the influence of two or more of these predictor variables together on yield, specifically, wheat density and wild oat density (Carlson et al. 1982; Chancellor and Peters 1974; Thurston 1962; Wilson et al. 1990); nitrogen rate and wild oat density (Bell and Nalewaja 1968; Bowden and Friesen 1968; Carlson and Hill 1985; Sexsmith and Russell 1963); nitrogen rate, wheat density, and wild oat density (Blackshaw et al. 2002, 2004; Carlson and Hill 1985; Farahbakhsh et al. 1987; Henson and Jordan 1982; Tollenaar 1992); nitrogen rate and available water (Campbell et al. 1993; Engel et al. 2001; Fernandez and Laird 1959; Henry 1971; Hunter 1958; Neidig and Snyder 1924; Racz 1974; Warder et al. 1963); soil moisture, wheat density, and wild oat density (Van Wychen 2002); and herbicide rate, wheat density, and wild oat density (Blackshaw et al. 2002; Van Wychen 2002). Other field studies have investigated the influence of topography, soil type, soil pH, gravimetric moisture content, and soil fertility (Dieleman 2000a, 2000b; Dille et al. 2002; Mortensen et al. 1993; Shatar and McBratney 1999) to predict weed occurrence and its influence on yield. However, no field studies exist that have explored the combined influence of nitrogen, herbicide, and available water specifically on spring wheat–wild oat interference.

In contrast to the previously described modeling and experimental studies, our objectives were (1) to use field and greenhouse data to determine the functional dependence between yield and the five predictors and (2) to use the structures determined in our first objective in conjunction with meta-analysis to parameterize the best-fitting model to the data for optimized decision making about agricultural inputs. Specifically, the long-term goal of this research was to select a model that was simultaneously highly predictive and biologically meaningful for incorporation into a decision support system that farmers and crop consultants can use to develop site-specific and variable management strategies for crop seeding, nitrogen, and herbicide rate.

To meet this goal, our strategy was to (1) gather as many data sets as possible where spring wheat yield was the dependent variable and where some of the five designated independent variables were included; (2) explore individual data sets using scatter plots and regression analysis such that important biological mechanisms were revealed, i.e., that all predictor interactions were exposed as well as each variable's influence on yield; (3) develop and parameterize a best-fitting empirical yield-prediction model from a candidate set of models based on the five selected variables using a combined data set created from the independent data sets; (4) conduct a five-variable greenhouse experiment to augment information from previous studies; (5) explore greenhouse data and update the prediction models to include as many of the predictor variables and their interactions as possible while adhering to the principle of parsimony (Burnham and Anderson 1998).

We have chosen these five specific predictors (i.e., wheat and wild oat density, nitrogen and herbicide rate, and growing-season precipitation/water) with the understanding that many other predictor variables could be considered and perhaps should be considered in future studies. We selected this collection of five predictor variables primarily for three reasons. First, an extensive literature search indicated these variables to be the five most influential on dryland wheat yield production. Second, adding more predictors would threaten model convergence. Third, these five variables are relatively easy for farmers to measure in comparison to predictors such as site-specific soil pH and moisture content, and three of the variables—wheat density (crop seeding rate), herbicide rate, and nitrogen rate—are variables that farmers can control. End-use applicability was of paramount importance when constructing our model sets.

Table 1. Collected spring-wheat data sets in Canada, United States, and Australia. All data sets (except no. 14) include corresponding growing-season precipitation values—one per year. X denotes measurements of the specified variable was collected at numerous random levels but did not have fixed treatment levels. All numbers in the Variables included columns denote the number of fixed treatment levels in each experiment.

					Variables included			
Set no.	Author	Year	Location	*n*	Crop density	Wild oat density	Nitrogen rate	Herbicide rate
1	Blackshaw and Molnar	1998 to 2001	Lethbridge, CAN	32	x	x	2	1
2	Blackshaw and Molnar	1998 to 2001	Lethbridge, CAN	62	x	x	1	3
3	Carlson and Hill	1978 to 1982	Davis, CA, USA	94	17	19		
4	Engel et al.	1996 to 1998	Havre, MT, USA	719	1		19	1
5	Jackson	1986, 1993 to 1996	Havre, MT, USA	72			18	
6	Lenssen	1998 to 2000	Big Sandy, MT, USA	89	x	x	x[a]	
7	Lenssen	1998 to 2000	Big Sandy, MT, USA	134	x	x	x[a]	
8	Lenssen	1998 to 2000	Box Elder, MT, USA	92	x	x	x[a]	
9	Lenssen	1998 to 2000	Box Elder, MT, USA	95	x	x	x[a]	
10	Martin and Riordan	1969	Tamworth, AUS	475	x	x		
11	Martin	1968, 1982 to 1983	Tamworth, AUS	25	x	x	1	1
12	Maxwell	1998 to 2001	Bozeman, MT, USA	413	x	x		
13	Murphy	1997 to 1999	Wagga Wagga, AUS	236	x	x		
14	O'Donovan et al.	1975 to 1976	Lacombe, CAN	44	1	x	1	1
15	Rew	1997 to 1999	Tamworth, AUS	490	x	x	1	1
16	Van Wychen et al.	1999 to 2000	Sun River, MT, USA	305	x	x	5	3
17	Van Wychen et al.	1999 to 2000	Sun River, MT, USA	218	x	x	5	3
18	Van Wychen et al.	1999 to 2000	Sun River, MT, USA	230	x	x	5	3

[a] Nitrogen treatment in this experiment was measured in terms of nitrate (ppm) in the top 2 feet of soil instead of kg N ha^{-1} applied.

Materials and Methods

Field Studies. Unfortunately, the ideal data set that includes all five predictor variables replicated at several levels and sites does not exist. Nevertheless, empirical models can be developed from the numerous on-farm and experiment-station trials that have been conducted for different purposes (Cousens et al. 1987). Thus, we have obtained and used data sets where subsets of the five specified variables were manipulated and measured. Studies that included several of the factors (e.g., spring wheat density, wild oat density, nitrogen rate, herbicide rate, and growing-season precipitation) in combination and at varying levels were chosen for our analysis. Experiment-station small-plot and on-farm large-plot data sets were accumulated from wheat regions of California; Minnesota; Montana; Alberta, Canada; and western Australia (Table 1).

We have used data sets in which wheat and wild oat densities were measured in quadrats at the seedling stage before herbicide application (if herbicide was applied). All density measurements were converted to the units of plants per square meter for uniformity across data sets. Yield was measured by a plot combine or by a farmer-owned combine with yield-mapping capabilities. Unlike the other four variables, which were typically an administered treatment within the respective experiment, growing-season precipitation (GSP) as a metric for available water, has also been included in all data sets because of the ease with which this information can be acquired from local weather stations. Inference from this metric was limited, however, because only one value could be obtained per year. GSP for U.S. and Canadian data sets were calculated by summing monthly totals of current-year precipitation from April through August. GSP for data collected in Australia was calculated by totaling monthly precipitation from April through October because of the increased length of the Australian growing season. We have used spring rainfall to calculate GSP, not including winter precipitation, assuming spring rainfall is most influential on crop yield.

Greenhouse Study. Although the previously described field data sets represent the combined efforts of a wide range of researchers over the past several decades, more data collection was necessary to increase certainty about the underlying ecological mechanisms of plant competition because none of these experiments covered the full range of factors and levels of our interest, such as more than one level of growing-season precipitation per year. Thus, the greenhouse study was undertaken, albeit with the understanding of the limitations of greenhouse studies because they do not completely mimic the environmental and plant dynamics in a field setting. The greenhouse experiment, nonetheless, had four major advantages over a field study. First, a full factorial five-variable experiment was possible. Second, extraneous factors were more easily controlled (e.g., soil type was held constant). Third, there was no confounding history of management (i.e., residual from previous years' applied fertilizer and herbicide and stored soil water). Finally, greenhouse experiments can allow for relatively quick replication compared with field experiments. The soil used in all replications consisted of two parts silt loam and one part washed concrete sand. To determine water treatments, a soil–water retention relationship was determined by drying soil samples over 2 wk to estimate hydraulic conductivity. Resulting measurements of gravimetric soil–water contents were fit to the Van Genuchten (1980) parametric equation to determine soil matrix potentials (Wraith et al. 1995). High, medium, and low soil–water content treatments were set up with estimated mass water content of the soil at 27.5, 19.7, and 17.0%, corresponding to soil matrix potentials of -0.1, -4, and -12 MPa respectively. A 17.8-cm-diam "standard azalea" pot size was used. Each pot was filled with 1,800 g of dry soil, sown with the corresponding densities of wheat and wild oat, and watered to field capacity for full germination. After germination of the wheat and wild oats, the soil in each pot was allowed to dry to the desired matrix potential. Pots were weighed every 2 to 3 d and watered to the desired percent water content. After watering, the soil was allowed to dry until

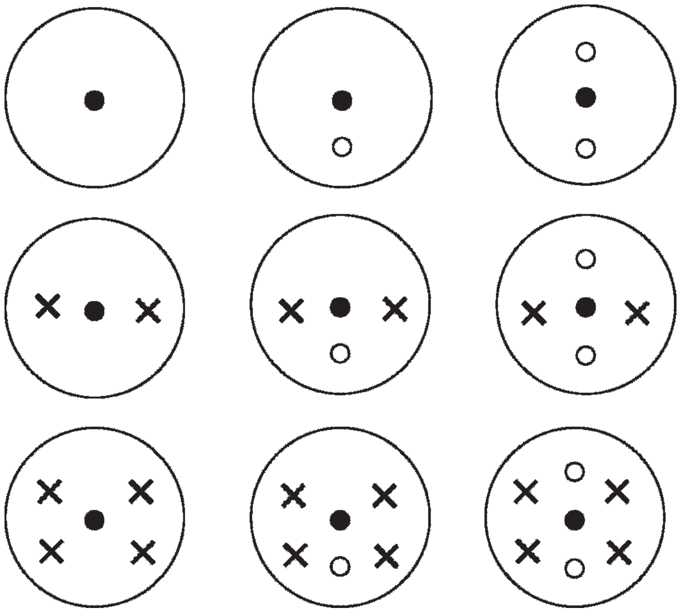

Figure 1. The nine spatial combinations of wheat and wild oat seeding are illustrated. Os represent wheat plants, and Xs represent wild oat plants. Darkened circles represent the central wheat plants that were analyzed for neighborhood intraspecific as well as interspecific competition.

the next watering. Pots were randomly arranged in the greenhouse and rotated every week to avoid edge effects and other area-specific effects of being in a greenhouse, such as continued placement in a shady or sunny area or near a cooling fan. When wheat reached the boot stage, approximately 32 to 35 d after sowing, wheat and wild oat plants were clipped at the soil surface and dried in a 50 C oven for 48 h, then weighed. Competition between wheat and wild oat plants was observed only until the boot stage to reduce the restrictive effects experienced by plants growing in pots. Dried wheat and wild oat plants from each pot were weighed to obtain biomass measurements. Plant biomass was assumed to have a linear relationship to yield (Cousens and Mortimer 1995).

Wheat and wild oat were planted in nine density and spatial arrangements (Figure 1). The three levels of wheat density (e.g., 1, 2, and 3 plants pot^{-1}) corresponded to field-planting densities of 170, 340, and 510 plants m^{-2}. Three levels of wild oat density were used to replicate a situation with no weed pressure, medium infestation, and high weed infestation. The three levels of wild oat density (e.g., 0, 2, and 4 plants pot^{-1}) corresponded to 0, 340, and 650 plants m^{-2}. All noncenter wheat and wild oat plants were sown 2.5 cm from the center wheat plant using cardboard templates. For each of the nine combinations of plant densities, each level of nitrogen, herbicide, and water was applied in combination for a total of 432 pots replication^{-1}. Ten days after planting, four rates of ammonium nitrate fertilizer were applied at 0, 22.5, 45, and 90 kg N ha^{-1}. At the two to four-leaf stage, imazamethabenz herbicide was applied at 0, 0.5×, 0.75×, and 1× the label rate. Biomass of the central wheat plant was compared across the crop–weed density treatments to elucidate the influence of competition as well as the influence of varying combinatorial levels of inputs on crop–weed competition.

Model Building and Statistical Analysis. Model development included the exploration of four historically used agronomic models and three of our development. The form of the three models we developed were based on the literature, specifically, the history of agronomic and competition models, and on patterns observed by making scatter plots and standardized regressions of the data. Scatter-plot analysis is the first fundamental step to model building because it visually reveals patterns in the data, such as linearity, nonlinearity, and interactions among independent variables (Cousens et al. 1987; Neter et al. 1996). Standardized regression analysis (Brown and Rothery 1993) quantifies the strength of each independent variable's influence on the dependent variable via the size of each variable's coefficient. This method of developing model form was based on the goals of obtaining a model that had properties consistent with historic models (e.g., Cousens 1985a), would account for interactions that allow for optimization of inputs, fit the wide range of data sets well, and would lend itself to future parameterization with field data sets with high variability.

The seven hypothesized candidate models were fit to the "combined" field data set (i.e., the data set that pooled all[1] independent data sets into one) and to the greenhouse data set to assess the predictability of the best-fitting models (see Table 2). Although combining data sets has been done previously by Cousens (1985b), Martin et al. (1987), and Tollenaar (1992) for investigating model variation, the history of combining agronomic data from different studies has been quite recent. Specifically, a more accurate amount of model variance can be revealed when there is a greater measured range of predictor and response variables, accounting for a greater representation of the entire response surface. Yield was the dependent variable in all seven models.

Model 1, called the rectangular hyperbolic model, included weed-free yield (y_{wf}) and weed density (ρ_w) as predictor variables, and two fit parameters (see Table 2). The parameter ι estimated proportional yield loss weed^{-1} at low weed density, and the parameter α estimated the asymptotic proportional yield loss at high weed density (Cousens 1985b). Model 1 was chosen for this analysis because it was shown to fit a large number of data sets more consistently than 17 other functional forms, as investigated by Cousens (1985b). Consequentially, the rectangular hyperbolic model gained acceptance for estimating crop yield response to varying densities of a single weed species (Swanton et al. 1999), as employed by Stoller et al. (1987), Wilson and Wright (1990), Weaver (1991), Coble and Mortensen (1992), Norris (1992), Sattin et al. (1992), Berti and Zanin (1994), and Lindquist et al. (1996).

Model 2, a version of the Beverton-Hold model (1957), included crop density (ρ_c) and weed density (ρ_w) as predictor variables, and the fit parameters r, b, and g (Baeumer and deWit 1968; Jollife et al. 1984; Weiner 1982; Wright 1981) (see Table 2). This form of the Beverton and Holt (1957) model, which includes interspecific competition, is a modification of the Hassel model for limited population growth in discrete time (Brown and Rothery 1993). In this case, r describes the crop's intrinsic growth rate, b was an intraspecific competition coefficient, and f was an interspecific competition coefficient. Specifically, as b and f increase

[1] Not every data set shown in Table 1 was included in the combined data set because at least one of the variable measurements was missing. All other independent data sets that included all five variables, even if a variable had only one rate, such as a broadcast nitrogen, herbicide, or crop seeding rate, were included in the combined data set.

Table 2. Hypothesized model forms for the seven candidate models.

Model no.		No. of estimable parameters
1	$\sqrt{y} = y_{wf}\left(1 - \frac{\iota\rho_w}{1 + \iota\rho_w/\alpha}\right)$ (Cousens 1985b)	3
2	$\sqrt{y} = \frac{r\rho_c}{1 + b\rho_c + f\rho_w}$ (Baeumer and deWit 1968; Wright 1981; Weiner 1982; Jollife et al. 1984)	3
3	$\sqrt{y} = y_{wf} \Big/ \frac{1 + \beta_{H_0}\rho_{w_0}}{1 + \left(\frac{H}{e^{H_{50}}}\right)^B}$ (Kim et al. 2002)	3
4	$\sqrt{y} = \left(\frac{\vartheta\rho_c}{1 + \vartheta\rho_c/y_{\max}}\right)\left(1 - \frac{\iota\rho_w}{1 + \iota\rho_w/\alpha}\right)$	4
5a	$\sqrt{y} = \left[\frac{\vartheta\rho_c}{1 + \vartheta\rho_c/(\beta_0 + \beta_1\sqrt{W} + \beta_2\sqrt{N} + \beta_{12}WN)}\right]\left[1 - \frac{\iota\rho_w}{1 + \iota\rho_w\Big/\left(\alpha_{\min} + \frac{\alpha_{\max} - \alpha_{\min}}{1 + e^{B(H - H_{50})}}\right)}\right]$	9
5b	$\sqrt{y} = \left[\vartheta\rho_c \Big/ \frac{1 + \vartheta\rho_c}{(\beta_0 + \beta_1\sqrt{W} + \beta_2\sqrt{N} + \beta_3\sqrt{H} + \beta_{12}WN + \beta_{13}WH)}\right]\left\{1 - \left[\iota\rho_w \Big/ \frac{1 + \iota\rho_w}{(\beta_{00} + \beta'_1\sqrt{W} + \beta'_2\sqrt{N} + \beta'_3\sqrt{H} + \beta'_{12}WN + \beta'_{13}\sqrt{WH})}\right]\right\}$	10
6	$\sqrt{y} = \vartheta\rho_c \Big/ \left\{1 + \left[\frac{\vartheta\rho_c}{(\beta_0 + \beta_1\sqrt{W} + \beta_2\sqrt{N} + \beta_3\sqrt{H} + \beta_{12}WN + \beta_{13}WH)}\right] + \left[\frac{\iota\rho_w}{1 + \iota\rho_w/(\beta_{00} + \beta'_1\sqrt{W} + \beta'_2\sqrt{N} + \beta'_3\sqrt{H} + \beta'_{12}WN + \beta'_{13}WH)}\right]\right\}$	10
7	$\sqrt{y} = \beta_0 + \beta_1 W + \beta_2\rho_w + \beta_3\rho_c + \beta_4 N + \beta_5 H + \beta_6 W^2 + \beta_7\rho_w^2 + \beta_8\rho_c^2 + \beta_9 N^2 + \beta_{10}H^2 + \beta_{11}WN + \beta_{12}WH + \beta_{13}NH + \beta_{14}W\rho_w + \beta_{15}W\rho_c + \beta_{16}\rho_w\rho_c$	17

due to intra- and interspecific competition, yield will decrease. Model 2 has had significant scientific input because it was the first to include competitive effects of both weed and crop density (Baeumer and deWit 1968; Jollife et al. 1984; Weiner 1982; Wright 1981).

Developed by Kim et al. (2002), model 3 included herbicide dose rate (H), the herbicide dose required to reduce the weed population by 50% (H_{50}), the response rate of the herbicide (B), initial weed density (ρ_{w0}), weed-free yield (y_{wf}), and weed competitiveness at zero herbicide dose (β_{H0}). Model 3 was included in this analysis because it was one of very few models in the literature that included an input (in this case, herbicide) as a predictor variable. To aid parameter convergence, H_{50} had an assumed value of 0.50.

Model 4, called the double hyperbolic model, included crop density (ρ_c), weed density (ρ_w), asymptotic maximum yield ($y_{\max}$), and the fit parameters φ, ι, and α (Jasieniuk et al. 2000). Specifically, φ estimated the initial rate of yield increase as crop density increases from zero, ι estimated the initial rate of yield loss as weed density increases from zero, and α is the asymptote for maximum percentage yield loss as weed density increases to its maximum. The first hyperbola described the nonlinear increase of yield as crop density increases. Yield increases to a maximum, which was defined by $y_{\max}$. The second hyperbola, based on the Cousens (1985b) rectangular hyperbolic model, described the nonlinear increase of yield loss as weed density increases to a maximum of α.

Model 5, an amended version of model 4, included the inputs of nitrogen, herbicide, and water. Model 5 was divided into model 5a and model 5b because they were essentially the same models, but the influence of herbicide rate and nitrogen rate on wild oats was incorporated into the two models slightly differently. Model 5a included growing-season precipitation (W), nitrogen rate (N), and their corresponding fit parameters. The size of the parameter values indicated the magnitude of influence water (β_1), nitrogen rate (β_2), and their interaction (β_{12}) had on the maximum yield in the field. The parameter $\alpha_{\min}$ was between 0 and 1 and described the minimum herbicide rate response, $\alpha_{\max}$ described the maximum herbicide dose response between 0 and 1, and B affected the slope of the curve. The component of model 5 that included these two parameters and herbicide rate was based on the herbicide dose–response equation of Streibig et al. (1993):

$$\alpha = \alpha_{\min} + \frac{\alpha_{\max} - \alpha_{\min}}{1 + e^{B(H - H_{50})}} \qquad [1]$$

where H_{50} was the herbicide rate (i.e., H_{50}) required to obtain a result half way between the upper limit, $\alpha_{\max}$, and the lower limit, $\alpha_{\min}$, on the herbicide dose rate response curve. On a log dose scale, the slope is maximal at the point $H = H_{50}$. Model 5b included the same variables as model 5a but did not include the sigmoidal herbicide dose rate equation (Equation 1). Realizing that the herbicide dose–response equation made model 5a quite complex, and thus difficult to fit, given the data sets in hand, model 5b included a simpler regression for the weed impact asymptote, including water and herbicide effects on wild oat (i.e., $\alpha = \beta_{00} + \beta_{11}\sqrt{W} + \beta_3\sqrt{H} + \beta_{13}\sqrt{WH}$).

Based on the double-hyperbolic model 4, models 5a and 5b were modified to include nitrogen rate and available water through the parameters β_0, β_1, β_2, and β_{12} (see Table 2). The size and sign (i.e., positive or negative) of each parameter indicated the influence of the corresponding variable on yield. The inclusion of these variables was made by regressing $y_{\max}$ (i.e., weed-free yield) on water level and nitrogen level. This $y_{\max}$ regression equation assumed water and nitrogen contributed to the maximum yield value in a field where there is little to no competition from weeds. Water (i.e., growing-season precipitation) and nitrogen were written as $\sqrt{W}$ and $\sqrt{N}$ because the square-root transformation of these variables provided the best fit. However, logarithm, natural logarithm, quadratic, and cubic transformations were explored as well. Given the difficulty of convergence of models 5a and 5b when fit to the combined field data, both models were reduced in form as discussed further in the Results section.

Because models 5a and 5b were relatively difficult to interpret, that is the multiplication of the two hyperbolas of the models were analytically complex, model 6 was developed (see Table 2). Model 6 made the same assumptions as the two previous double-hyperbolic models. Model 6, however, was an alternative functional form with more tractable mathematical properties. Additionally, model 6 was developed for possible increased potential for parameter convergence. Model 6 included the intrinsic growth rate of the crop (φ). All other variables and parameters are previously defined. Instead of splitting the effects of crop density and weed density on yield into two hyperbolas, model 6 added the effect of crop density, as influenced by water (β_1), nitrogen (β_2), and herbicide (β_3), and their interactions (β_{12} and β_{13}), to the effect of weed density, as influenced by water (β_1'), nitrogen (β_2'), and herbicide (β_3'), and their interactions (β_{12}' and β_{13}').

Model 7 was included in the analysis to contrast the fit of a simple multiple linear-regression model with the other nonlinear models (see Table 2). Model 7 assumed that all main effects and their interactions were additive. If model 7 fit the data as well or nearly as well as the other complex nonlinear models, evidence for nonlinear ecological patterns and interactions would be lacking in support. All five main effects were included in model 7. The hypothesized nonlinear effects of wheat density, wild oat density, and nitrogen rate on wheat yield were included via their squared terms. All interactions between the five variables were explored, as indicated in Table 2, but not all were statistically significant as explained further in the Results section.

Exploring the normality of residuals of the models was accomplished by investigating residual vs. fit, response vs. fit, and residual Normal Q–Q plots in S-PLUS.[2] Residuals of the models fit to the combined data set were shown to be normal after the square-root transformation of the dependent variable (i.e., yield) was used in the model. Because the residuals were normally distributed for all models after this transformation was made, the least-squares (LS) method, as opposed to Fisher's maximum-likelihood (ML) method, was used to find the best-fitting parameter estimates. S-PLUS was used to calculate the least squares. The square-root transformation has been used previously for wheat yield modeling (O'Donovan et al. 1985, 2005). Although these two methods do not yield identical squared standard error ($\hat{\sigma}^2$) values for linear and nonlinear models because ML and LS estimators differ by a factor of $n/(n - p + 1)$, the difference is slight given our large sample size of 1,627 points (Burnham and Anderson 1998). Mean-squared errors ($\hat{\sigma}$), R^2 values, Akaike Information Criteria (AIC), and Bayesian Information Criteria

(BIC) model-selection statistics were used for comparing these nonnested models (Burnham and Anderson 1998), such that the lowest AIC and BIC values denoted the best-fitting models. BIC was used in addition to AIC because it penalizes overfitting (e.g., using more model parameters) more severely. Given the large number of observations, however, conclusions made from AIC and BIC statistics were in agreement.

Results

Model Selection Based on Field Data. Because of the lack of convergence in models 5a and 5b to the combined field data, models 5a and 5b were reduced to the same form:

$$\sqrt{y} = \left(\frac{\vartheta\rho_c}{1 + \vartheta\rho_c/(\beta_0 + \beta_{1W})}\right)\left(1 - \frac{\iota\rho_w}{1 + \iota\rho_w/\alpha}\right) \qquad [2]$$

As with models 5a and 5b, the model-selection statistics indicated that the data did not warrant inclusion of certain parameters in model 6 as hypothesized (see Table 3). The best-fitting version of model 6 to converge with parameter estimates was

$$\sqrt{y} = \frac{\vartheta\rho_c}{1 + \left(\frac{\vartheta\rho_c}{\left(\beta_0 + \beta_1\sqrt{W} + \beta_2\sqrt{N}\right)}\right) + \left(\frac{\iota\rho_w}{1 + \iota\rho_w/\left(\beta_{00} + \beta_1'\sqrt{W} + \beta_3'\sqrt{H}\right)}\right)} \qquad [3]$$

According to backward step-wise regression, where terms significant at $P < 0.10$ remained in the equation, model 7 was reduced, including the 12 parameters listed in Table 4.

The lowest ΔAIC value (Table 4) showed that the best-fitting model was the multiple linear-regression model, closely followed by a reduced version of models 5a and 5b (model 5), as revealed by its lowest residual standard error. The ranges of ΔAIC values were quite large partly because of the large sample size. To visually examine goodness-of-fit for each model, response (observed yield) values vs. fit (predicted yield)

Table 3. Summary of model-selection statistics ($n = 1{,}627$) for model fits to the combined field data. An asterisk (*) next to estimates indicate parameters whose values were set at the indicated number to allow nonlinear least-squares convergence. The ΔAIC of the best-fitting model is 0. Each model's ΔAIC value was calculated by taking the difference between its AIC value and the AIC value of the best-fitting model.[a]

	Parameter Values					
Model	Fit parameters	Estimates	P value	ΔAIC	R^2	MSE $\hat{\sigma}$
1	ι	0.02	< 0.0001	1,459	0.210	14.9
	y_{wf}	56.2	< 0.0001			
	α	0.50	< 0.0001			
2	r	26.6	0.2402	1,458	0.208	14.9
	b	0.47	0.2475			
	f	0.27	0.2474			
3	β_{H0}	0.001	0.0021	2,156	0.0	18.5
	B	2.7	< 0.0001			
	y_{wf}	40*				
4	y_{wf}	58.6	< 0.0001	1,508	0.096	15.1
	φ	7.1	< 0.0001			
	ι	0.01	< 0.0001			
	α	1*				
5	ι	0.003	< 0.0001	238	0.628	10.2
	φ	0.002	< 0.0001			
	α	0.45	< 0.0001			
	β_0	133.3	< 0.0001			
	β_1	1.38	< 0.0001			
6	ι	0.174	< 0.0001	472	0.564	11.1
	φ	1.73	< 0.0001			
	β_0	25.0	< 0.0001			
	β_1	5.75	< 0.0001			
	β_2	1.36	< 0.0001			
	β_{00}	2.44	< 0.0001			
	β_1	−0.15	< 0.0001			
	β_3	2.72	0.0002			
7	β_0(incpt)	10.42	< 0.0001	0	0.705	9.0
	β_1(water)	−0.12	0.0021			
	β_2(wo)	−0.10	< 0.0001			
	β_3(wheat)	0.32	< 0.0001			
	β_4(nitro)	−0.08	0.0121			
	β_5(herb)	−25.4	< 0.0001			
	β_7(wo^2)	0.0001	< 0.0001			
	β_8(wheat^2)	−0.0003	< 0.0001			
	β_9(nitro^2)	0.0003	< 0.0001			
	β_{11}(water : nitro)	0.0039	< 0.001			
	β_{12}(water : herb)	0.55	< 0.001			

[a] Abbreviations: ΔAIC, Akaike Information Criteria; MSE, mean squared error; ι, proportional yield loss per weed at low density; y_{wf}, weed-free yield; α, asymptotic proportional yield loss at high weed density; r, the crop's intrinsic growth rate; b, an intraspecific competition coefficient; f, an interspecific competition coefficient; β_{H0}, weed competitiveness at zero herbicide dose; B, response rate of the herbicide; φ, estimated the initial rate of yield increase as crop density increases from zero; and crop density, as influenced by β_0, intercept in first hyperbola; β_1, water; β_2, nitrogen; β_{00}, intercept in second hyperbola; β_3, herbicide; β_0(incpt), intercept in first hyperbola; β_1(water), water; β_2(wo), wild oat; β_3(wheat), wheat; β_4(nitro), nitrogen; β_5(herb), herbicide; β_7(wo^2), wild oat; β_8(wheat^2), wheat; β_9(nitro^2), nitrogen; β_{11}(water : nitro), the interaction of water and nitrogen; and β_{12}(water : herb), the interaction of water and herbicide. The squared terms refer to coefficients of non-linear variables.

Table 4. Summary of model-selection statistics (n = 1,244) for each model's fit to the greenhouse data where wheat biomass (approximately 35 d after emergence) was the dependent variable. An asterisk (*) indicates a parameter whose estimate was set at the indicated value to allow convergence.[a]

Model	Parameter Values: Fit parameters	Estimates	P values	ΔAIC	R^2	MSE ($\hat{\sigma}^2$)
1	ι	0.0007	0.0009	1,161	0.117	0.256
	α	0.48	0.0032			
	y_{wf}	0.94	< 0.0001			
2	r	0.013	< 0.0001	871	0.306	0.227
	b	0.011	< 0.0001			
	f	0.002	< 0.0001			
3	β_{H0}	0.0005	< 0.0001	1,513	0.0	0.295
	y_{wf}	0.65	< 0.0001			
	B	0.9*				
4	ι	0.006	< 0.0001	876	0.305	0.228
	φ	0.010	< 0.0001			
	α	0.52	0.0035			
	y_{wf}	1.35	< 0.0001			
5a	ι	0.001	0.0153	14	0.651	0.161
	φ	0.009	< 0.0001			
	β_0	−0.37	< 0.0001			
	β_1	0.06	< 0.0001			
	β_2	0.015	0.0041			
	β_{12}	−0.0000	0.0014			
	α	0.68	0.0020			
5b	ι	0.001	0.0106	15	0.652	0.161
	φ	0.009	< 0.0001			
	β_0	−0.24	0.5834			
	β_1	0.05	< 0.0001			
	β_2	0.0008	0.8555			
	β_{12}	−0.0002	0.0035			
	β_{00}	0.96	0.0007			
	β_1'	−0.013	0.0231			
	β_3'	−0.053	0.1835			
	β_{13}'	0.0004	0.0150			
6	ι	0.005	< 0.0001	0	0.656	0.160
	φ	1.07	< 0.0001			
	β_0	−29.3	< 0.0001			
	β_1	3.26	< 0.0001			
	β_2	0.096	0.5419			
	β_3	−2.67	0.0754			
	β_{00}	12.2	0.0191			
	β_1'	−0.182	0.0322			
	β_2'	0.147	0.1021			
	β_3'	−4.02	0.1034			
7	β_0	0.056	0.2365	57	0.651	0.168
	β_1(water)	0.0005	< 0.0001			
	β_2(wo)	0.0005	< 0.0001			
	β_3(wheat)	0.002	< 0.0001			
	β_4(nitro)	0.002	0.0005			
	β_5(herb)	−0.100	0.0241			
	β_6(wheat^2)	0.0000	< 0.0001			
	β_7(wo^2)	0.0000	0.0001			
	β_9(nitro^2)	0.0000	0.0033			
	β_{11}(nitro : water)	0.0002	0.0003			
	β_{12}(water : herb)	−0.001	0.0053			
	β_{14}(wo : water)	0.0000	0.0100			

[a] Abbreviations: ΔAIC, Akaike Information Criteria; MSE, mean squared error (i.e., residual standard error); ι, proportional yield loss per weed at low density; α, asymptotic proportional yield loss at high weed density; y_{wf}, weed-free yield; r, the crop's intrinsic growth rate; b, an intraspecific competition coefficient; f, an interspecific competition coefficient; B_{H0}, weed competitiveness at zero herbicide dose; B, response rate of the herbicide; φ, estimated the initial rate of yield increase as crop density increases from zero; and crop density, as influenced by β_0, intercept in first hyperbola; β_1, water; β_2, nitrogen; β_{12}, and their interactions; β_{00}, intercept in second hyperbola; the effect of weed density, as influenced by β_1', water; β_2', nitrogen; β_3', herbicide; β_{13}', and their interactions; β_1(water), water; β_2(wo), wild oat; β_3(wheat) wheat; β_4(nitro), nitrogen; β_5(herb), herbicide; β_6(wht^2), wheat^2, wheat; β_7(wo^2), wild oat; β_9(nitro^2), nitrogen; β_{11}(nitro : water), the interaction of nitrogen and water; β_{12}(water : herb), the interaction of water and herbicide, and β_{14}(wo : water), the interaction between water and wild oat. The squared terms refer to coefficients associated with non-linear variables.

values were plotted (Figure 2). Models 5, 6, and 7 provided far better wheat-yield predictability than the other four models (Figure 2).

A high level of model complexity was supported by fitting the models to the combined data set even though the scatterplot analysis of individual data sets and the combined data set revealed a great deal of variability and asymptotic yield as inputs were increased. Specifically, all five predictor variables in models 6 and 7 were shown to be significantly (P < 0.10) influential on yield. Despite the assumption in model 7 that all main effects, squared main effects, and interactions were additive, this model was able to capture interactions previously found in the literature—specifically between nitrogen and water (Campbell et al. 1993; Engel et al. 2001; Henry et al.

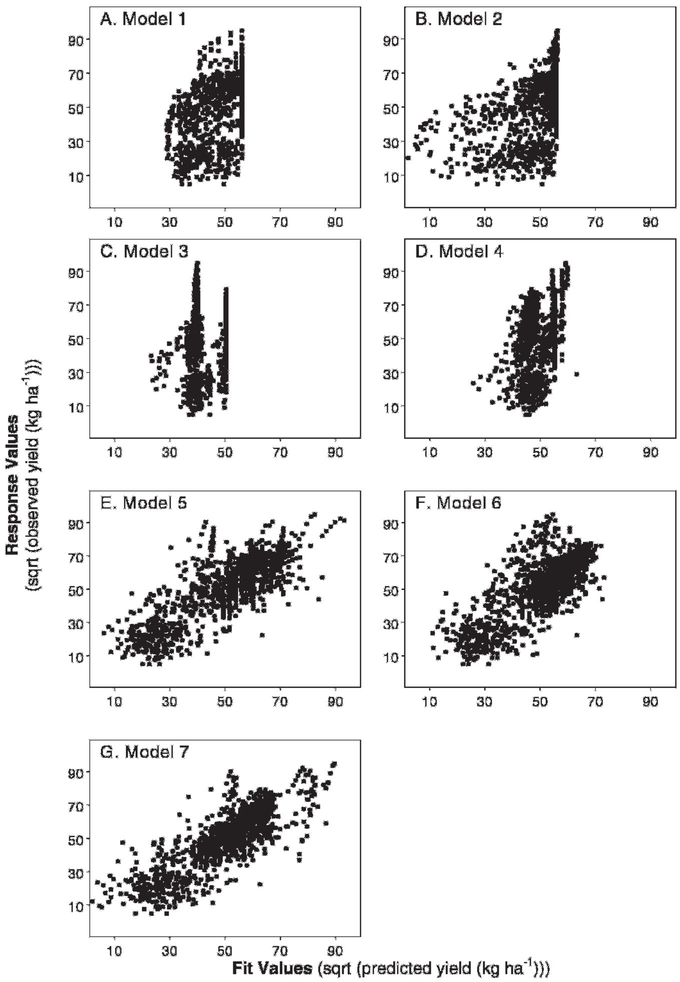

Figure 2. Scatter plots showing goodness-of-fit for each model to the combined field data.

1986) and between herbicide and water (Grundy et al. 1996). Model 7, albeit a linear model, also supported the asymptotic behavior of wheat density and wild oat density on yield. However, despite the good fit produced by model 7, coefficients for GSP and nitrogen rate were negative, implying, counterintuitively, that these treatments had negative influences on wheat yield. Model 7 has the disadvantage that it does not draw upon decades of agronomic research that have established first principle nonlinear responses of wheat yield to the five variables studied here (Bell and Nalewaja 1968; Bowden and Friesen 1967; Martin et al. 1987; O'Donovan et al. 1985, 2005; Wilson and Peters 1982).

In contrast, model 5, the second-best-fitting model, did include underlying nonlinear yield responses with parameters that are of the correct sign, such as an estimate of the initial rate of yield increase as crop density increases from zero, an estimate of the initial rate of yield loss as weed density increases from zero, and the asymptote for maximum proportional yield loss as weed density increases. Additionally, model 5 supported a greater level of complexity than previously developed nonlinear plant competition models by including GSP as a predictor variable. Inference from GSP is limited, however, because only one GSP value was obtained per year, so that it was impossible to separate this variable from generalized year and location effects.

Model 6, although not producing as low an AIC value as model 5 nor as high an R^2 value, converged with the inclusion of nitrogen and herbicide rate as well as GSP. The predicted vs. observed yield plots show goodness-of-fit to the combined data as compared with previously developed agronomic models (Figure 2). Although the modeling results showed advancement in the area of yield prediction, the sizable variance in the data, as revealed by the large deviation among points in the scatter-plot analysis and the low R^2 values in the standardized-regression analysis, is cause for further inquiry.

Our collection of worldwide data sets represents the combined efforts of an entire discipline over the past several decades and forms the basis for the best-fitting three and five-variable models presented in this article. Inferences were limited because none of the data sets measured all five predictor variables at more than one level. Thus, assumptions were necessary and sources of variation were overlooked. For example, soil type was not included in the models. Rather, soil type was essentially treated as uniform across the fields (i.e., not contributing to wheat yield) where experiments were conducted. There are many soil quality factors that could cause variation in wheat yield response to wild oats, e.g., nitrogen, herbicide, and water. Additionally, relative time of emergence, which was not included in the vast majority of data sets, was assumed the same for wheat and wild oat for our model development purpose. Although the fit of Models 5 and 6, in particular, is a remarkable step in yield-prediction modeling, we conducted the greenhouse experiment to further investigate the variability in wheat–wild oat systems, thus allowing further five-variable model development.

Model Selection Based on Greenhouse Data. Model 5a did not reach convergence when herbicide was added to the model via the herbicide dose–response equation. Therefore, $\alpha_{\min} + \left[(\alpha_{\max} - \alpha_{\min})/1 + e^{B(H - H_{50})}\right]$ was reduced to α for convergence, and the total number of estimable parameters became seven (Table 4). Models 5a, 5b, and 6 (Table 4) included the transformations for water, nitrogen, and herbicide (e.g., $\sqrt{W}$, $\sqrt{N}$, and $\sqrt{H}$) found to provide the best-fit and biological reality. The nitrogen–water and herbicide–water interaction terms in models 5a, 5b, and 6 were not square-root transformed because it produced a worse fit. Because of difficulties in convergence of parameter estimates when all terms were included in these models as hypothesized in Table 2, they were reduced in form. Models 5a, 5b, 6, and 7 were reduced to the following forms, respectively:

$$\sqrt{y} = \left(\frac{\vartheta\rho_c}{1 + \vartheta\rho_c/\left(\beta_0 + \beta_1\sqrt{W} + \beta_2\sqrt{N} + \beta_{12}WN\right)}\right)\left(1 - \frac{\iota\rho_w}{1 + \iota\rho_w/\alpha}\right) \quad [4]$$

$$\sqrt{y} = \left(\frac{\vartheta\rho_c}{1 + \vartheta\rho_c/\left(\beta_0 + \beta_1\sqrt{W} + \beta_2\sqrt{N} + \beta_{12}WN\right)}\right)\left(1 - \frac{\iota\rho_w}{1 + \iota\rho_w/\left(\beta_{00} + \beta_1'\sqrt{W} + \beta_3'\sqrt{H} + \beta_{13}'WH\right)}\right) \quad [5]$$

$$\sqrt{y} = \vartheta\rho_c \div \left[1 + \left(\frac{\vartheta\rho_c}{(\beta_0 + \beta_1\sqrt{W} + \beta_2\sqrt{N} + \beta_3\sqrt{H})}\right) + \left(\frac{\iota\rho_w}{1 + \iota\rho_w/(\beta_{00} + \beta_1'\sqrt{W} + \beta_2'\sqrt{N} + \beta_3'\sqrt{H})}\right)\right] \quad [6]$$

$$\sqrt{y} = \beta_0 + \beta_1 W + \beta_2\rho_w + \beta_3\rho_c + \beta_4 N + \beta_5 H + \beta_6\rho_c^2 + \beta_7\rho_w^2 + \beta_8 N^2 + \beta_9 NW + \beta_{10} WH + \beta_{11}\rho_w W \quad [7]$$

Parameter estimates for models 5a, 5b, 6, and 7 (Equations 4 to 7) are given in Table 4.

As revealed by its lowest ΔAIC and mean squared error ($\hat{\sigma}^2$) values and highest R^2 value, the best-fitting model was model 6 (Table 4). Model 6 included available water and nitrogen effects on wheat biomass, and water and herbicide effects on wild oat biomass. Models 5a, 5b, and 7 provided reasonable fits according to their ΔAIC, mean squared error (MSE), and R^2 values. Models 5a and 5b had the second-lowest ΔAIC and MSE values. Models 5 to 7 fit the data similarly well and were strikingly better fits of the data than models 1 to 4 (Figure 3). Additionally, fits of models 5 to 7 to the data showed less variance than the fits of models 1 to 4 (Figure 3). Specifically, the two best-fitting models, models 5b and 6,

- Estimated the initial rate of yield increase as crop density increased from 0 (by the parameter φ)
- Estimated the initial rate of yield loss as weed density increased from 0 (by the parameter ι)
- Implied asymptotic behavior of yield vs. wheat density and yield vs. wild oat density
- Showed that water level, nitrogen rate, and herbicide rate significantly affect wheat yield (model 5b only showed water level and nitrogen rate to significantly affect wheat yield)
- Showed interactions between nitrogen rate and water level and herbicide rate and water level (model 5b only), and
- Indicated that nitrogen rate, herbicide rate, and water level have nonlinear effects on wheat yield.

According to supplemental biomass measurements taken of wheat and wild oat plants across treatments, wheat appears to outcompete wild oat for water. Parameter estimates derived from the fit of model 6 to the greenhouse experiment seem to parallel this finding, although Martin and Field (1998) indicated that wild oat was the better root competitor. Also based on biomass measurements, nitrogen had a positive effect on wheat and wild oat but had a slightly larger positive effect on the wild oat plants, indicating that wild oat outcompetes wheat for nitrogen across water and herbicide treatments. Revealed by the sign of its parameter estimates (e.g., $\beta_3 = -2.67$ and $\beta_3' = -4.02$, Table 4), herbicide had a negative effect on both wild oat and wheat plants across all nitrogen and water levels, but its negative effect on wild oat was nearly two times as great in magnitude. Although the literature supports the interactions between nitrogen and water (Campbell et al. 1993; Engel et al. 2001; Henry et al. 1971) and herbicide and water (Grundy et al. 1996), they were not supported by the fit of model 6 to the greenhouse data because these interactions were not statistically significant in the model at the $P < 0.10$ level. The fit of model 5b to the greenhouse data did support the inclusion of these interactions.

Although the independently combined field data did not show nitrogen's influence on wild oat density to be significant, fitting models 5 to 7 to the greenhouse data revealed that nitrogen could have significant positive influence on wild oat density. Thus, the greenhouse data results supported past literature results and revealed additional inference over the combined field data; this substantiated the value in further greenhouse experimentation where all five variables can be measured in factorial combination. Given that none of the field data sets included all five variables measured at three or more levels, inference was limited.

When fitting the two components of model 6 (e.g., $(\beta_0 + \beta_1\sqrt{W} + \beta_2\sqrt{N} + \beta_3\sqrt{H})$ and $(\beta_{00} + \beta_1'\sqrt{W} + \beta_2'\sqrt{N} + \beta_3'\sqrt{H})$) to the greenhouse data, there was evidence of improved fit (e.g., improved R^2, MSE, and AIC values) using square-root transformations of the variables water, nitrogen, and herbicide (i.e., W, N, and H, respectively). The square-root transformation implied nonlinearity of the effects these three predictor variables have on yield. Model 5a implied that the nonlinear effects of herbicide

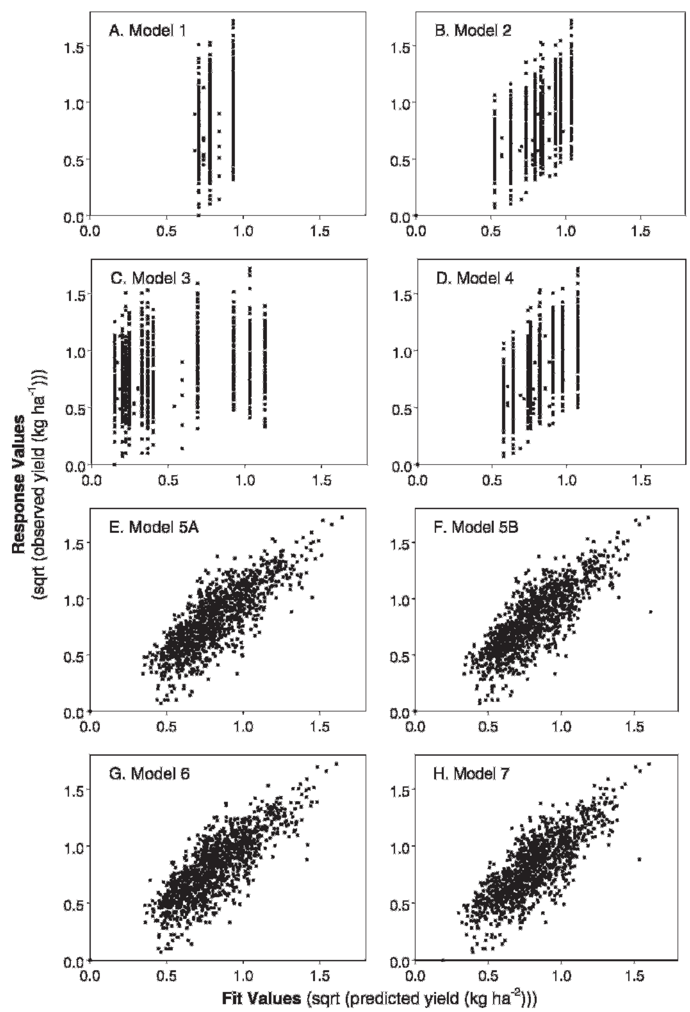

Figure 3. Scatter plots showing goodness-of-fit for each model to the greenhouse data set.

on wild oat density via the herbicide dose curve (Streibig et al. 1993) were not shown to be significant. Given that only four herbicide rates were included in our experiment, however, the sigmoidal dose–response curve would be difficult to fit. For such a curve to be fit, at least six rates would be required.

To determine possible improvement of net return using site-specific recommendations as output by the best-fitting model, we propose that model 6, the best-fitting nonlinear model of the candidate set of models, be used for optimization of localized nitrogen and herbicide rates. Although model 7 provides a very good fit of the data, model 6 was favored over model 7 because it incorporated forms and known trends revealed in historical agronomic models. Optimization of inputs throughout a field is possible using early season wheat and wild oat seedling densities and localized soil moisture values as input values to explore how well the output of model and parameter estimates represents localized, variable-rate management strategies. Such a demonstration would reveal the direction for further research in this area, specifically involving the execution of studies on farm and agricultural experiment stations.

In summary, the main contribution of this work was to identify a first-principle model that included the agronomic variables that can be controlled with management (crop seeding rate, nitrogen rate, herbicide rate) and a set of variables that naturally cause variation (weed density, water level) in crop yield. The goodness-of-fits revealed sizeable potential for the advancement of localized, variable-rate input management using precision agriculture technologies via a decision support system including first-principle models like 5 to 7. We propose that model 6, the best-fitting model of the candidate set of models, be used for optimization of localized nitrogen and herbicide rates on farms using parameter estimates obtained from the greenhouse study and the independently collected field data. Model 6 may be more accurately predictive on farms if it included parameter estimates completely derived by field data; however, the field data did not allow for the convergence of model 6. Therefore, to use model 6 in a field application, a starting point is to use the combination of best-fit greenhouse and field parameter estimates. After the initial growing season, model 6 could be updated with all site-specific field parameter estimates. Subsequent growing seasons would allow for continual improvement of parameter estimates and model form, as well as model validation. Such a demonstration would reveal the value in using a nonlinear, five-variable yield model and the development of site-specific management strategies.

Sources of Materials

[2] S-PLUS. 2002. S-PLUS, Insightful Corporation, Seattle, WA.

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