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Journal of Hydrology 324 (2006) 40–50
Journal of Hydrology
www.elsevier.com/locate/jhydrol

Soil hydraulic properties near saturation, an improved conductivity model

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Received 20 July 2004; revised 25 August 2005; accepted 8 September 2005

Abstract

The hydraulic properties near saturation can change dramatically due to the presence of macropores that are usually difficult to handle in traditional pore size models. The purpose of this study is to establish a data set on hydraulic conductivity near saturation, test the predictive capability of commonly used hydraulic conductivity models and give suggestions for improved models. Water retention and near saturated and saturated hydraulic conductivity were measured for a variety of 81 top and subsoils. The hydraulic conductivity models by van Genuchten [van Genuchten, 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. 44, 892–898.] (vGM) and Brooks and Corey, modified by Jarvis [Jarvis, 1991. MACRO—A Model of Water Movement and Solute Transport in Macroporous Soils. Swedish University of Agricultural Sciences. Department of Soil Sciences. Reports and Dissertations 9.] were optimised to describe the unsaturated hydraulic conductivity in the range measured. Different optimisation procedures were tested. Using the measured saturated hydraulic conductivity in the vGM model tends to overestimate the unsaturated hydraulic conductivity. Optimising a matching factor (k_0) improved the fit considerably whereas optimising the l-parameter in the vGM model improved the fit only slightly. The vGM was improved with an empirical scaling function to account for the rapid increase in conductivity near saturation. Using the improved models, it was possible to describe both the saturated and the unsaturated hydraulic conductivity better than a previously published model by Jarvis. The pore size boundary of the macropores was found at a capillary pressure of −4 hPa corresponding to a circular pore diameter of 750 μm.

Keywords: Near saturated hydraulic conductivity; Macropores; Optimisation method

1. Introduction

Detailed information on the soil hydraulic properties close to saturation is important to simulate transport of water and chemicals in soils. Different models to describe the soil unsaturated hydraulic conductivity from soil water retention characteristics are available and have been evaluated by Alexander and Skaggs (1986); Leij et al. (1997).

Often, the saturated hydraulic conductivity (k_s) is measured in field scale water-flow studies, whereas measured unsaturated hydraulic data near saturation

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doi:10.1016/j.jhydrol.2005.09.014

often are extrapolated from k_s using hydraulic conductivity models. Vogel and Cislerova (1988) found for conductivity models based on soil water retention curves, that the inaccuracies in the determination of the retention curve had a large influence on the prediction of hydraulic conductivity especially near saturation. Vogel et al. (2001) introduced a modified version of the van Genuchten Mualem model (vGM) van Genuchten (1980), extending the retention model with an boundary pressure head h_s as the minimum capillary height and a empirical parameter for water content at h_s. The effect of the modified vGM model on prediction of the soil hydraulic functions near saturation on numerical water flow predictions were evaluated. This study concluded that for especially fine-textured soils, simulations of water infiltration were highly sensitive to small errors in the predictions of the hydraulic conductivity curve shape near saturation and they recommended the use of the modified version of the vGM hydraulic conductivity model for fine-textured soils.

van Genuchten and Nielsen (1985); Luckner et al. (1989) argued that using k_s in the conductivity model by van Genuchten (1980) (VGM) can lead to an overestimation of the unsaturated hydraulic conductivity due to the effect of macropores on the measured saturated hydraulic conductivity. They recommended using the hydraulic conductivity measured close to saturation as matching point.

The models by van Genuchten (1980); Brooks and Corey (1964) reflect unimodal pore size distribution. Multidomain models are being developed (Gerke and van Genuchten, 1993; Hutson and Wagenet, 1975; Durner, 1994; Mohanty et al., 1997) to describe this non-ideal pore size distribution effect on the soil hydraulic properties. Multimodal models are often based on a number of underlying submodels of unimodal type derived from different pore domains. To parameterise this type of model, detailed information on the soil hydraulic properties is required.

In simulations of water and solute transport at both plot, field and large scale (region, catchment) it is often not possible to parameterise multidomain hydraulic models representing different soil types, due to lack of detailed soil hydraulic measurement. Here a more simple approach is needed to improve the description of the hydraulic properties. Near saturation, the unsaturated conductivity often decreases very rapidly with decreasing water content caused by active macropores and cracks. To describe this rapid decrease in hydraulic conductivity, Jarvis (1991) presented an equation based on the Brooks and Corey (1964) retention model and the Mualem (1976) model for hydraulic conductivity. This model assumes that the pore system is split up into non-interacting macropores and micropore domains described by different retention and hydraulic conductivity models. The conductivity in the Jarvis model often changes sharply at the interconnection between the two pore domains, which is physically unrealistic.

The objectives of this study are therefore: (i) to test the predictive capability of commonly used hydraulic models in the near saturated range ($h > -70$ hPa) using different optimisation strategies, and (ii) to set up and parameterize an improved hydraulic conductivity model to predict near saturated hydraulic conductivity.

1.1. Theory

Burdine (1953); Mualem (1976) presented pore-size distribution models to estimate the unsaturated hydraulic conductivity from pore size distributions inferred from soil water retention characteristics. The models share a number of similarities, allowing them to be written in a general form as (Hoffmann-Riem et al., 1999)

$$k(S_e) = k_0 S_e^l \left(\int_0^{S_e} |h|^{-\beta} \, dS_e / \int_0^1 |h|^{-\beta} \, ds_e \right)^{\gamma} \tag{1}$$

where k is the unsaturated hydraulic conductivity (cm day^{-1}), S_e is the effective saturation, h the pressure head (hPa). The parameter k_0 (cm day^{-1}) is a matching factor, often taken as the saturated hydraulic conductivity (k_s). Generally accepted parameter values for the l, β, and γ parameters are (2, 2, 1) and (0.5, 1, 2) for the Burdine (1953); Mualem (1976) models, respectively. In this study we will focus on the Mualem variant, as it is the most widely used in soil science and hydrology.

A closed-form of (1) can be obtained by expressing h in terms of S_e. To this end, van Genuchten (1980) proposed the following water retention model.

$$S_e = \frac{\theta-\theta_r}{\theta_s-\theta_r} = \begin{cases} \frac{1}{[1+|\alpha h|^n]^m}, & h<0, \\ 1, & h \geq 0, \end{cases} \quad (2)$$

where α and n are curve shape parameters and $m = 1-1/n$. The parameters θ, θ_s and θ_r are the actual, saturated and residual water contents, respectively. Based on (1) and (2), the following hydraulic conductivity model is obtained:

$$k(S_e) = k_0 S_e^l [1-(1-S_e^{1/m})^m]^\gamma \quad (3)$$

Using k_s as a matching factor is not a good choice (van Genuchten and Nielsen, 1985; Luckner et al., 1989). Hoffmann-Riem et al. (1999); Schaap and Leij (2000) found that using $k_0 < k_s$ obtained far better fits than using $k_0 = k_s$ and $l = 0.5$. However, Schaap and Leij (2000) also noted that a model with fitted k_0 cannot provide a realistic description of conductivity near saturation. A sudden discrete decrease from k_s to k_0 at $h = 0$ hPa is clearly unrealistic and leads to ambiguity as to which value should be used. Schaap and Leij (2000) demonstrated that (3) with fitted k_0 systematically underestimates hydraulic conductivity between 0 and -10 cm. However, in comparison with the 'standard' van Genuchten-Mualem approach (with $k_0 = k_s$ and $l = 0.5$), their approach gives a better description of conductivity at more negative pressures.

As shown by Hoffmann-Riem et al. (1999); Schaap and Leij (2000), the parameter l plays a critical role in the vGM model in describing the unsaturated hydraulic conductivity. Hoffmann-Riem et al. (1999) argued that to get a reasonable fit using this model, the parameter values were often not physically meaningful. They recommended that besides k_0, the parameters l and γ should be optimised. To reduce the number of free parameters in the models and avoid strong correlation between the fitted curve shape parameters, only the k_0 and the l parameter in (3) are optimised on the conductivity data measured in this study.

To describe the rapid decrease in hydraulic conductivity near saturation Jarvis (1991) presented retention and hydraulic conductivity models partly based on the model set-up by Brooks and Corey (1964); Mualem (1976). The model is a bi-domain pore system consisting of a macropore dominated system and a matrix pore dominated system and is described by two interconnected functions. The retention model is expressed as

$$S_e = \begin{cases} \left(\frac{\theta-\theta_r}{\theta_b-\theta_r}\right) = \left(\frac{h}{h_b}\right)^\lambda, & h \leq h_b, \\ \left(\frac{\theta-\theta_b}{\theta_s-\theta_b}\right) = \left(1-\frac{h}{h_b}\right), & h > h_b, \end{cases} \quad (4)$$

where h_b is the boundary pressure head and θ_b is the boundary water content. Notice that the air entry value in the original Brooks and Corey model does not equal h_b in the Jarvis model. To compute the actual water content using (4), the water content at h_b (θ_b) has to be fitted together with, respectively, θ_s and θ_r. This gives the model of five retention parameters that can be fitted to the measurements. The hydraulic conductivity model by Jarvis (1991) is expressed as

$$k(S_e) = \begin{cases} k_b (S_e)^n, & h \leq h_b, \\ k_b + (k_s - k_b)(S_e)^{n*}, & h > h_b, \end{cases} \quad (5)$$

where $n = 2 + 2\frac{1}{2}\lambda$ following Brooks and Corey (1964); Mualem (1976), n^* is an empirical parameter (in this study the n^* parameter was fixed at 5 as originally proposed by Jarvis, 1991) and k_b is the boundary hydraulic conductivity at h_b. The hydraulic conductivity in the macroregion is simply based on an empirical exponential function connecting k_b and k_s.

We propose a new empirical scaling function to scale the conductivity functions to give a realistic curve shape in the near saturated region:

$$p_m = \begin{cases} \left(\frac{1}{|h|\chi+1}\right)^f, & h > h_m, \\ \left(\frac{1}{|h_m|\chi+1}\right)^f, & h \leq h_m, \end{cases} \quad (6)$$

The h_m (hPa) parameter can be interpreted as the boundary between pressure heads at which macropore flow or matrix flow dominates. $\chi = 1$ hPa^{-1} is a constant to make the scaling factor dimensionless. Without priory knowledge on pore size boundary between the two domains, the h_m parameter is a fitting parameter and the f-parameter is a curve shape parameter of the conductivity function. A combination of the vGM conductivity model (3) and (6) gives

$$k(S_e) = k_0 S_e^l [1-(1-S_e^{1/m})^m]^\gamma p_m \quad (7)$$

The model is referred to as vGMP (7). p_m is constant for $h<h_m$, giving predictions similar to optimising k_0 in the vGM model (3). By introducing the scaling function we achieve that (i) the vGMP model becomes smooth at the interconnections at $h=h_m$, and (ii) the vGMP model always starts at the measured saturated hydraulic conductivity.

2. Material and methods

2.1. Soil data set

Soil samples from two or three horizons (depth 0–30, 30–70 and 70–140 cm, respectively) have been sampled and analysed in 32 Danish soil profiles representing mainly the textual classes sand, sandy loam and loam. In this study, the soils were classified into three soil classes (Clayey, Loamy, and Sandy). The textural composition of the 81 soil horizons is shown in Fig. 1. The figure also shows the textural classes of the soils divided into the USDA textural classification system as well as the textural classes adopted in this study.

Soil water retention was measured at 6–7 pressure heads (−10, −32, −50, −100, −400, −1000, −15,000 hPa) on five undisturbed soil cores (100 cm^3). The arithmetic mean of the five measurements was used in this study. At pressure heads −10, −32, −50 and −100 hPa, we used a sandbox and hanging water column. At −400, −1000, and −15, 000 hPa, a pressure chamber with ceramic plates was used. At −15,000 hPa the water content was determined on disturbed soil samples. Porosity (ϕ) is calculated from bulk density (BD) assuming an average density of soil of 2.65 g cm^{-3}. Undisturbed cylindrical 20 cm×20 cm columns were sampled for hydraulic conductivity measurements with two replications for each horizon.

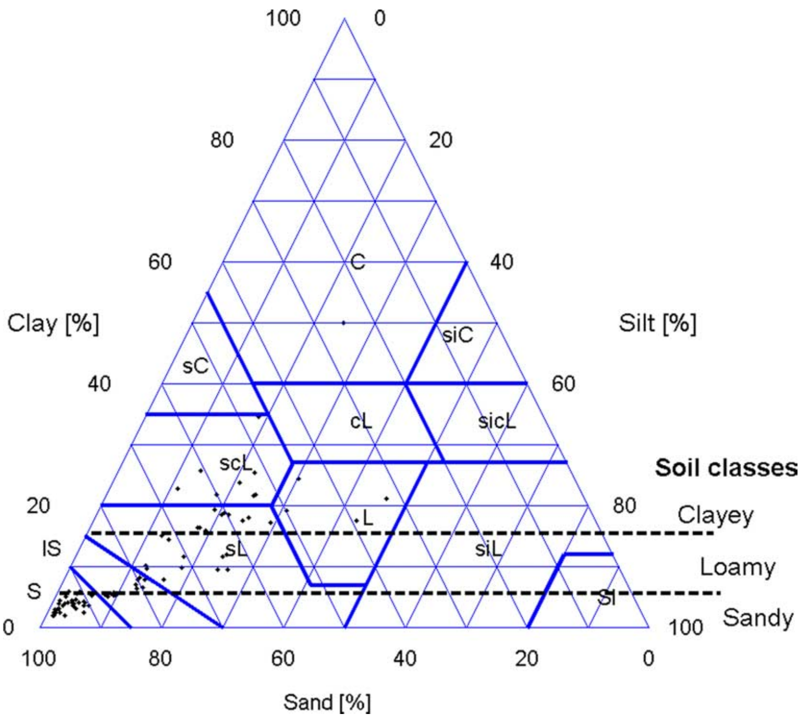

Fig. 1. Textural distribution of the 81 soils classified into the USDA textural classification system. Soil data used in this study were classified into three soil classes (clayey, loamy and sandy).

Table 1
Number of horizon observations and the total number of measurement points obtained within the three soil classes

Soil class	Horizon observations	Retention points	Conductivity points
Sandy	27	212	387
Loamy	28	202	533
Clayey	26	177	341

Saturated hydraulic conductivity was measured using the constant head method and the unsaturated hydraulic conductivity (near saturation > -70 hPa) was measured using a drip infiltrometer (van den Elsen et al., 1999). The principle is: a water saturated soil sample is placed on a sandbox with adjustable suction ranging from -10 to -150 hPa: water is applied on top of the sample using a needle device. The pressure head at five depths of the soil sample is continually measured using ceramic cups connected to transducers. When steady-state water flow condition in the soil and a pressure head gradient of less than 1 hPa cm^{-1} has been reached, a measurement is conducted. The unsaturated hydraulic conductivity is calculated using the Darcy equation on the basis of the flux density and the gradient measured through the soil sample. Table 1 gives the summarised number of measurement points of retention data and hydraulic conductivity obtained within each soil class.

2.2. Data analysis

2.2.1. Hydraulic parameters

To get a continuous description of the water retention curve on horizon level, the van Genuchten retention model (2) and the Jarvis model (4) were fitted to the data using the Levenberg–Marquardt algorithm (Marquardt, 1963). Different combinations of initial parameter values were used in the optimisation to ensure that the global minimum had been reached. The objective function of the optimisation of the retention parameters is

$$\mathrm{SSR_W} = \sum_{i=1}^{N} (\theta_i - \theta_i')^2 \tag{8}$$

where θ_i and θ_i' are the water contents measured and predicted, respectively, and N is the number of measured water retention points for each sample (horizon). To avoid unreasonable values, the following constraints were imposed on the vGM-parameters: $0 \le \theta_r < 0.5*\phi$, $0.8*\phi < \theta_s < 1.2*\phi$, $0.0 < \alpha < 0.1$ (hPa^{-1}), $1.001 < n < 10$, $0.01 < k_0 < 10*k_s$. Notice that θ_s is allowed to exceed ϕ because of the uncertainty due to estimating the porosity from bulk density. For the Jarvis parameters, the following constraints were used: $0 \le \theta_r < 0.5*\phi$, $0.8*\phi < \theta_s < 1.2*\phi$, $0 > h_b \ge -20$ (hPa), $k_b \le {}_s k_s$.

In the optimisation of the hydraulic conductivity models logarithmic transformation was used to have variance homogeneity. The objective function of this optimisation is

$$\mathrm{SSR}_k = \sum_{i=1}^{N} (\mathrm{Log}\ 10(k_i) - \mathrm{Log}\ 10(k_i'))^2 \tag{9}$$

where k_i and k_i' are the unsaturated conductivity measured and predicted, respectively, and N is the number of measured data points of the unsaturated hydraulic conductivity for each sample (horizon).

To avoid bias of the standard conductivity models, it was found necessary to allow higher k_0 values than the measured k_s for approx. 40% of the soils. The lower constraint was never exceeded; the upper constraint ($10*k_s$) was necessary for approx. 10% of the soils. To ensure a continuously decreasing $k(S_e)$ with decreasing water content, constraints were imposed for the l parameter (Table 2). In the different optimisation, strategies constraints were imposed on k_0.

Four strategies (M1, M2, M3, and M4, Table 2) to optimise the conductivity parameters for the standard conductivity models were examined in order to find the optimal combination of hydraulic conductivity model and parameter estimation strategy. In M1 the unsaturated conductivity was predicted using the measured saturated hydraulic conductivity as a matching factor (k_0). In strategy M2 we optimised k_0 on the measured unsaturated conductivity measurements using the originally proposed values for $l = 1/2$, whereas in M3 we optimised k_0 using a generally optimised l value. The generally optimised l value was found by obtaining the lowest Root Mean Square Residuals for conductivity measurements (RMSR_k) (10) using a stepwise optimisation approach similar to the approach used in Schaap and Leij (2000). They optimised the l parameter on data from the UNSODA

Table 2
Parameter estimates (fixed or fitted) and constraints used in the four optimisation strategies (M1, M2, M3, M4) of the different hydraulic conductivity models

Model	Optimisation strategies				General constraints
	M1	M2	M3	M4	
vGM	$k_0=k_s$, $l=1/2$	K_0=fitted, $l=1/2$	K_0=fitted, $l=-1$	K_0=fitted, l=fitted	$10k_s>k_0>0.01$ cm h^{-1}, $-2-2/(n-1)<l<100$
vGMP	–	h_m=fitted, f=fitted, $l=1/2$	–	h_m=fitted, f=fitted, l=fitted	$h_m>0, f>0, -2-2/(n-1)<1<100$, $k_0=k_s$
Jarvis	–	k_b=fitted, $n^*=5$	–	k_b=fitted, n^*=fitted	$k_s \geq k_b > 0.01$, $n^*>0$

database Leij et al. (1996) and found better predictions (lower RMSR_k) using $l=-1$ in the Mualem model instead of $l=1/2$ as originally proposed by Mualem (1976). In our study, similar optimisation was conducted. The l parameter was optimised on the data set by stepwise (step of 0.5), changing the l value in the range [−5…5], and then optimising k_0.

The vGMP model (7) and the Jarvis model (5) were optimised using two strategies, M2 and M4, respectively (Table 2). We note that different constraints have been used in optimising the improved model (vGMP) compared to the standard hydraulic conductivity model (vGM) hence the results of similar optimisation strategies are not directly comparable.

2.2.2. Evaluation procedure

The $\text{RMSR}_{k,\text{w}}$ for the water retention and unsaturated hydraulic conductivity data and the average deviation (DEV_k) for only the hydraulic conductivity were computed to evaluate random and systematic errors:

$$\text{RMSR}_{k,\text{w}} = \sqrt{\frac{\text{SSR}_{k,\text{w}}}{N}} \tag{10}$$

$$\text{DEV}_k = \frac{1}{N}\sum_{i=1}^{N}(\text{Log }10(k_i) - \text{Log }10(k_i^l)) \tag{11}$$

In (10) the residual sum of squares (SSR) is divided by the number of observation (measurement points), N, to avoid assigning larger weight for soils with large N values (Clausnitzer et al., 1992).

3. Results and discussion

3.1. Retention parameters

An overview of average retention parameter values and standard deviations is listed in Table 3. The parameter values and the average RMSR_w are presented to the three adopted textural soil classes and for all 81 soils. For the vGM model, we found individually RMSR_w between 0.005 and 0.060 cm^3 cm^{-3}. The RMSR_w varied between the soil classes showing the highest values (poorest fit) for the sandy (S) and lowest for loamy and clayey samples. The Jarvis model had low RMSR_w values for the sandy and loamy but high values for the clayey samples. In the Jarvis model a lower constraint for h_b was introduced, which forced the air entry $h_b \geq -20$ hPa. This constraint was introduced to avoid predictions of unsaturated hydraulic conductivity in the matrix using the macropore function. For clayey soils a free fitted air entry value is often lower than −100 hPa. The curve shape of the Jarvis model combined with the constraint for the air entry value h_b ($h_b \geq -20$ hPa) enables the model to make good predictions for the clayey soil types. The two functions in the Jarvis model (4) have independent parameters describing the curve shape in the wet and dry region of the retention curve. This was found to give the model a higher degree of flexibility (lower RMSR_w) in fitting to retention data for especially sandy soils. For the soils as a whole, it was found that the vGM-model described the retention data best. The reason why the two domain models of Jarvis do not fit the water retention of the macroporous clayey soil better than the vGM model, supports the general

Table 3
Average hydraulic parameters (standard deviation in parentheses) obtained for each soil class

Model	α and h_b^{-1} (hPa^{-1})	n and λ	θ_r	θ_s	Average $RMSE_w$
Sandy (n=27)					
VGM	0.056 (0.021)	1.686 (0.256)	0.027 (0.024)	0.442 (0.046)	0.042 (0.018)
Jarvis	0.058 (0.015)	0.753 (0.354)	0.010 (0.009)	0.405 (0.354)	0.025 (0.011)
Loamy (n=28)					
VGM	0.038 (0.028)	1.487 (0.223)	0.018 (0.024)	0.446 (0.059)	0.025 (0.010)
Jarvis	0.056 (0.015)	0.377 (0.223)	0.013 (0.021)	0.437 (0.050)	0.030 (0.013)
Clayey (n=26)					
VGM	0.042 (0.081)	1.393 (0.157)	0.005 (0.015)	0.403 (0.069)	0.017 (0.007)
Jarvis	0.055 (0.013)	0.1591 (0.088)	0.0027 (0.009)	0.409 (0.067)	0.052 (0.027)
All soils (n=81)					
VGM	0.045 (0.050)	1.521 (0.250)	0.017 (0.023)	0.431 (0.061)	0.028 (0.016)
Jarvis	0.055 (0.012)	0.161 (0.086)	0.003 (0.009)	0.408 (0.066)	0.035 (0.021)

All parameters were optimised on retention data only.

findings, that while conductivity can vary by orders of magnitude with a small change in pressure head close to saturation, the water content will be much less affected, e.g. Mohanty et al. (1997); Clothier and Smettem (1990).

3.2. Hydraulic conductivity model predictions

Overviews of average values of $RMSR_k$ and DEV_k are listed in Table 4.

We found that the error of the predictions was highly dependent on the optimisation strategy. The poorest fit was found using M1 where $RMSR_k$ for all soils varied between 0.16 and 3.57 for the vGM. The average $RMSR_k$ was 1.23 and the average DEV_k was −0.52 corresponding to an average overprediction of the hydraulic conductivity of about a factor of 3. The results varied between soil classes and the best fits were found for the sandy and loamy samples. For the clayey soils, the average $RMSR_k$ had the same numeric values as the average DEV_k. Hence, the error is primarily caused by an overestimation of the hydraulic conductivity. In similar optimisations using k_s as matching factor, Schaap and Leij (2000) found an overprediction of the unsaturated hydraulic conductivity of one order of magnitude.

As expected, the effect of optimising k_0 in M2 reduced the $RMSR_k$ considerably and reduced the DEV_k to near zero. The average $RMSR_k$ obtained in the stepwise optimisation approach for the l parameter in the vGM model is shown in Fig. 2. The results show an average minimum of $l=-1$ for all soils. Compared with the results found for $l=1/2$ as originally proposed by Mualem (1976), we found only a minor improvement (lower $RMSR_k$) in the predictions using $l=-1$. As our data set only has measurements in the wet h-range $h>-70$ hPa and the results obtained for the UNSODA-data set (Schaap and Leij, 2000) represents data for a larger h-range, the results are not direct comparable but

Table 4
Statistics comparing the log10 transformed, observed, and vGM predicted hydraulic conductivity for combinations of soil classes and optimisation methods (M1..M4)

	Soil classes						All	
	Sandy		Loamy		Clayey			
	$RMSR_k$	DEV	$RMSR_k$	DEV	$RMSR_k$	DEV	$RMSR_k$	DEV
M1	0.78	0.04	0.95	−0.39	1.98	−1.96	1.23	−0.52
M2	0.40	0.02	0.40	0.01	0.44	0.14	0.41	0.05
M3	0.41	0.00	0.37	0.00	0.39	0.08	0.39	0.02
M4	0.32	0.00	0.36	0.01	0.38	0.07	0.37	0.04

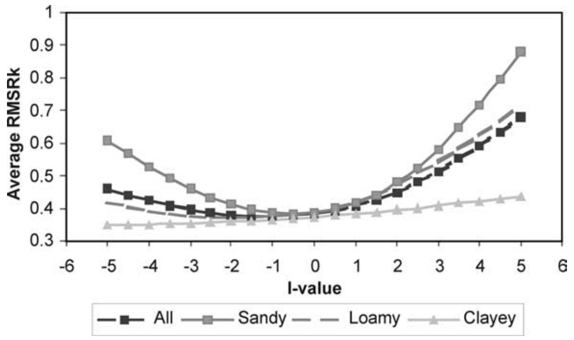

Fig. 2. Average $RMSR_k$ found for the three soil classes and for all soils optimising l in the van Genuchten–Mualem hydraulic conductivity model.

generally gave similar results. The improvement using $l=-1$ in M3 was found for the loamy and clayey soil classes (Table 3). In M4 we optimised both k_0 and the l parameter for each horizon.

Optimisation strategy M4 gave the best fit to the data of the evaluated methods. The improvement in average $RMSR_k$ was especially pronounced for sandy, whereas the improvement was considerably lower for loamy and clayey soils. We found that for some soils the optimised l parameter in the vGM models could not be used for extrapolation to lower water content due to a physically unrealistic curve shape in the h-range outside the measurement range. This limits the predictive capability of this optimisation strategy.

3.3. Improved hydraulic conductivity model predictions

In order to overcome the problems of fitting the van Genuchten model both to saturated hydraulic conductivity and to unsaturated hydraulic conductivity close to saturation we proposed an improved vGM model. It was found that combining vGM (3) with the empirical scaling function (6) gave a high degree of flexibility in fitting data and generally a low $RMSR_k$. The results of the improved model were compared with the Jarvis model. An overview of the results for the different optimisation strategies is shown in Table 5. Fig. 3 shows an example of curves of the vGMP model and the Jarvis model fitted to data representing a sandy subsoil. The vGMP curve is smooth at h_m, whereas the Jarvis curve changes more sharply at h_b, due to using two independent functions in the macro and matrix h-range.

In many cases k_s is mainly determined by a few large pores, and is poorly reflected by the general pore size distribution. The advantage of the vGMP and the Jarvis model is that it is more flexible than other conductivity models in the prediction of both the saturated and the unsaturated hydraulic conductivity. The vGMP and the Jarvis model all perform better than vGM using measured k_s (M1) (Tables 4 and 5, respectively). The better performance is especially evident for the clayey soils that also have the greatest tendency to have stable macropores.

Strategy M4 performed better than M2. For all soils, the improvement in $RMSR_k$ for the Jarvis model equals approx. 0.1 and for the vGMP model it was 0.13. The improvements for the vGMP model were primarily found for the sandy soil class and for the Jarvis model for loamy and clayey soils.

We note that the parameters f and h_m in the improved hydraulic model (7) were optimised on the hydraulic conductivity data, whereas in the Jarvis model k_b was optimised on the hydraulic conductivity data and h_b was optimised on the retention data. The improved models have thereby one more degree of

Table 5
Statistics comparing the log10 transformed, observed, and computed hydraulic conductivity for combinations of soil classes and optimisation method

	Model	Soil classes						All	
		Sandy		Loamy		Clayey			
		$RMSR_k$	DEV	$RMSR_k$	DEV	$RMSR_k$	DEV	$RMSR_k$	DEV
M2	vGMP	0.59	0.33	0.48	0.17	0.48	0.19	0.52	0.23
	Jarvis	0.43	−0.04	0.55	−0.06	0.69	−0.36	0.56	−0.15
M4	vGMP	0.37	0.09	0.37	0.04	0.42	0.15	0.39	0.09
	Jarvis	0.42	−0.01	0.49	0.01	0.49	−0.13	0.46	−0.05

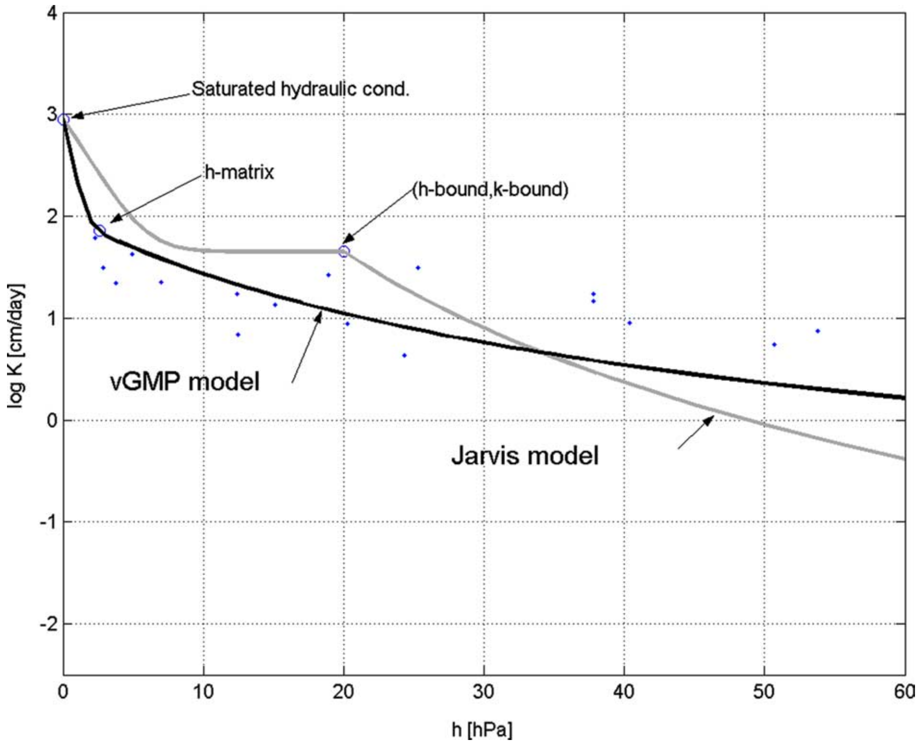

Fig. 3. The Jarvis model and the improved van Genuchten–Mualem model (vGMP) calibrated (method M2) to the saturated and unsaturated hydraulic conductivity measurements for a sandy subsoil (3% clay). *h*-matrix is the vGMP parameter h_m, both *h*-boundary and *k*-boundary are the Jarvis parameters h_b and k_b, respectively.

freedom in the optimisation to the measurements, which results in lower RMSE than found for the Jarvis model.

Table 6 gives the average values of the fitted Jarvis parameters and of the fitted parameters used in the improved model vGMP. The number of soils classified within each soil class and the number of soils that had optimised h_m values lower than 0 hPa are also shown, whereas the soil with a $k_b < k_s$ is shown for the Jarvis model. For these soils the scaling function improved the predictions. The percentage of soils scaled increased from approx. 55% for sandy to 100% for clayey soils indicating that for all the clayey soils the measured saturated hydraulic conductivity would overestimate the unsaturated conductivity using the vGM model.

The average results for the improved models represent only parameter values for the proportion of

Table 6
Number of soils scaled within each soil class for method M4, of the improved model (vGMP) and the Jarvis model

Soil classes	Number of soils	vGMP			Jarvis		
		Number of soils $h_m < 0$ hPa	h_m	f	Number of soils $k_b < k_{sat}$	h_b	N^*
Sandy	27	15	−4.1 (1.2)	1.2 (0.7)	22	−18.1 (3.0)	146 (271)
Loamy	28	22	−4.2 (1.1)	1.3 (2.2)	23	−19.6 (3.0)	63 (99)
Clayey	26	26	−4.2 (1.9)	2.7 (1.7)	26	−18.8 (3.0)	27 (28)
All	81	63	−4.1 (1.0)	1.9 (1.9)	71	−18.8 (2.6)	76 (146)

Average values of the parameters and their standard deviation (shown in parentheses).

soil that has been scaled within each soil class. The results for the improved models are found using method M2 and for the Jarvis model using M4. The average h_m parameter does not change between the soil classes (Table 6) indicating that $h = -4$ hPa as a plausible boundary between pressure heads dominated by matrix or macropore flow regimes. This boundary is purely based on whether the unsaturated hydraulic conductivity could be described from the soil water retention characteristics. Beven and German (1982) listed different size-boundaries for macropores. Luxemore (1981) found a boundary size of 1000 μm based on soil water transport characteristics. Our results, based on similar characteristics, show $h = -4$ hPa—corresponding to a pore diameter of 750 μm—is a general value for the pore-size boundary for macropores. The f parameter—on the other hand—increases from approx. 1.1 for sandy to 2.7 for clayey soils showing a larger average decrease in conductivity near saturation for the more fine-textured soils.

4. Conclusion

Optimising the van Genuchten retention model to our retention data set gave average Root Mean Square Residuals ($RMSR_w$) of 0.028 $cm^3\ cm^{-3}$. The Jarvis model gave a higher average $RMSE_w$ for all soils, but performed better than the vGM model for sandy soils.

When optimising both vGM-parameters (both k_0 and the l parameter on the unsaturated conductivity measurements) the lowest $RMSR_k$ (best fit) was found.

Improving the vGM model with an empirical scaling function made it possible to describe the unsaturated hydraulic conductivity and still fit the saturated hydraulic conductivity measured. For all the three soil classes examined, on average the scaling function improved the predictions from $h \cong -4$ hPa to saturation. These results suggest that a general pore size of 750 μm is a general value for the pore size boundary for macropores.

Due to the small number of measurements close to saturation ($h > -5$ hPa), it was not possible to examine the validity of the curve shape of the empirical model in this region. In general, the improved vGMP model performed better than the model proposed by Jarvis (1991).

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