Abstract: Irrigation practitioners use empirical infiltration equations. Theoretical infiltration equations are currently not capable of capturing surface irrigation infiltration behavior, particularly during initial wetting. For a coarse textured soil, an example is shown where the Green-Ampt equation can be adjusted to match field "average" infiltration conditions by altering the soil's physical properties. For finer textured soils, a time offset is proposed for adjusting the Green-Ampt equation to account for cracking and soil consolidation upon wetting. This results in a nonzero infiltration amount at time 0, a phenomenon commonly observed for infiltration of cracking soils. Applying this concept to the Philip equation (same as Modified Kostiakov equation with a=1/2) suggests the addition of an offset parameter. A modification to the two-point method is presented for this equation with the aim to better fit mathematical parameter functions to infiltration data.

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Introduction

Despite advances in the estimation of infiltration from soil physical properties (e.g., Simunek et al. 1999), surface irrigation practitioners still use empirical infiltration equations. In hydrologic modeling, the Green-Ampt equation has been used because of its simplicity and relative accuracy (Rawls et al. 1993). For landscapes dominated by rangeland, forests, grasslands, etc., such an approach has proven useful. Tillage practices on crop land have a significant influence on infiltration since they loosen or compact the soil. Once the soil is consolidated from irrigation or rainfall or wheel compaction, infiltration becomes more predictable. In arid areas where irrigation is practiced, soil cracking is common, which has a significant influence on soil infiltration. The influence is greater under surface irrigated conditions than under rainfall or sprinkler irrigation. With rainfall, initial wetting is generally slow, which might allow the cracks to close before runoff occurs. Under surface irrigation, a large stream of water quickly fills the cracks, allowing much more infiltration in a short time. To date, it has not been possible to adjust the inputs to soil-physics-based infiltration models to match observed infiltration conditions under surface irrigation. Except for a few references discussed here, we could find no other literature that shows substantial progress in fitting actual surface irrigation data to theoretical infiltration equations.

The purpose of this article is to propose a new approach to adjusting infiltration models for application to surface irrigation. The approach also suggests a general strategy for adjusting empirically based infiltration equations, including the development of a new two-point method for estimating infiltration from advance data. The approach here considers "average" infiltration conditions for a given irrigation event and does not explicitly address spatial and temporal variation in infiltration conditions.

Theoretical Infiltration

Philip (1957) provided explicit, yet theoretically based infiltration equations. The equation is the solution to the one-dimensional (1D) Richards infiltration equation for a soil with homogeneous physical properties and water content, in which water potential varies with water content. The solution takes the form of a power series in the infiltration time \( \tau \) with terms \( \tau^{1/2}, \tau, \tau^{3/2}, \ldots \). Philip (1957) proposed using the first two terms in this series as an explicit infiltration solution at short times

\[
D = S\tau^{1/2} + A\tau
\]

where \( D \) = infiltrated depth; \( S \) = sorptivity (a function of water content and diffusivity); and \( A \) = constant related to the saturated hydraulic conductivity \( K_s \). The first term on the right hand side is the complete solution to horizontal infiltration, or infiltration in the absence of gravity. This is the rate that water is taken into the soil due to capillary uptake or sorption (Philip 1957), and behaves as a function of \( \tau^{1/2} \). For a homogenous soil, the exponent \( 1/2 \) represents the smallest exponent on infiltration-time for cumulative infiltration into the soil, since all other terms have larger exponents. The gravitational contribution behaves as a function of larger powers of \( \tau \) (i.e., 1, 3/2, 2 etc.). Thus according to this theoretical solution, a power law relationship for infiltration can only have exponents equal to or greater than 1/2.

Eq. (1) is only valid for short times and Philip (1966) suggested that infiltration approaches an asymptote at long times described by
where \( G \) = empirical constant that incorporates the initial infiltration. At intermediate times, the actual solution lies between results from these two equations. Hartley (1992) discussed these limitations and provided equation adjustments and typical parameter values to transition between these two equations.

The Richards equation provides a more exact solution of the general infiltration problem and can deal with the influence of variations in soil conditions with depth. However, a numerical solution is required to solve for infiltration, which can be complex and time consuming. Numerous experiments have been conducted with Hydrus 2D (Simunek et al. 1999), an infiltration model based on the Richards equation. For example, Furman et al. (2006) used the Hydrus 2D model to determine 1D infiltration which they then fit to empirical infiltration equations. Ahuja et al. (2007) determined 1D infiltration for eleven soil-texture-classic soils using the Green-Ampt model, and then showed that:

1. the parameters of the Lewis-Kostiakov empirical infiltration equation are related to and scaled with \( K_r \) or pore-size distribution index (\( \lambda_m \)) across all soil textural classes; and
2. the Lewis-Kostiakov equation can be extended from instantaneous ponded-water infiltration to sprinkler and rain infiltrations. Warrick et al. (2007) used Hydrus 2D to compare 1D infiltration with two-dimensional (2D) infiltration for furrows, from which they determined an adjustment to 1D infiltration that accounts for 2D effects. While the effect of geometry on infiltration is important, it is not explicitly considered herein. Wohling et al. (2004) and Zeri-hun et al. (2005) have developed models that couple Hydrus 2D to simulation of surface irrigation flow. The current effort is aimed at adjusting infiltration model inputs to match observed conditions, not linking surface and subsurface flow.

Many hydrologic models have chosen to describe infiltration with the much simpler Green-Ampt equation, which can be solved analytically in most cases. The Green-Ampt solution assumes that there is an abrupt wetting front, rather than assuming a diffuse front where the water potential varies with water content. Bouwer (1969) suggests the following expression in terms of time

\[
\tau = \frac{\Delta \theta}{K_e} \left[ L - (h_s - h_f) \ln \left( \frac{L + h_s - h_f}{h_f - h_s} \right) \right]
\]

where \( \Delta \theta \) = difference in soil water content (saturated minus initial); \( K_e \) = effective hydraulic conductivity (typically 1/2 the saturated hydraulic conductivity); \( L \) = location of the wetting front from the soil surface; \( h_s \) = water depth on the soil surface; and \( h_f \) = pressure head at the wetting front (a negative value representing capillary forces). The infiltrated depth is \( \Delta \theta \).

Bouwer (1969) applied this equation to multiple soil layers with different initial water contents. Because it is difficult to directly solve these equations for multiple soil layers, he developed a simplified procedure based on superimposition of time. First, the time to infiltrate water into the first layer is calculated \( \tau_1 \). Then assuming both soil layers have the water content of the second layer, the time to infiltrate both the first and second layers are calculated, \( \tau_1 \) and \( \tau_2 \). The time for water to infiltrate through the second layer, \( \Delta \tau_2 = \tau_2 - \tau_1 \). The process is repeated for each layer. When only the soil moisture content varies between layers, this superimposition of time gives exactly the same solution as the equation derived for this case, namely

\[
\Delta \tau_2 = \frac{\Delta \theta}{K_e} \left[ (L_2 - L_1) - (h_s - h_f) \ln \left( \frac{L_2 + h_s - h_f}{h_f - h_s} \right) \right]
\]

The same procedure was also used for layers with different hydraulic conductivities, but then average (harmonic mean) hydraulic conductivity for the layers being considered was used. For nonuniform hydraulic conductivity, the method does not properly integrate average hydraulic conductivity with depth, and thus is only approximate (Gill 1977). However, this superimposition has important implications for the work that follows. When infiltration starts into the second layer, the time it took for water to infiltrate through the first layer no longer has any influence on the rate of infiltration into the second layer and through the soil surface. Thus, infiltration rate is a function of current conditions, independent of past history.

### Empirical Infiltration Equations

A simple power function (Kostiakov or Kostiakov-Lewis equation) has been used to describe 1D vertical infiltration

\[
D = k \tau^a
\]

where \( D \) = depth infiltrated; \( \tau \) = infiltration opportunity time; and \( k \) and \( a \) = constants. If \( D \) is in millimeters and \( \tau \) is in hours, then \( k \) has units mm/h\(^a\). The parameter \( a \) is unitless. Use of this relationship was practical for use with ring infiltrometer data since coefficients could be determined from a straight line on logarithmic paper. Additional terms are often added to this equation to provide for a constant final infiltration rate. The Modified Kostiakov equation is often of the form

\[
D = k \tau^a + b \tau
\]

where \( b \) has units mm/h and represents the final infiltration rate. With this formulation, \( b \) is not actually the final infiltration rate during the irrigation, since the exponential terms does not go to 0 quickly. To avoid this difficulty and simplify the analysis, the branch infiltration function was developed (Clemmens 1981, following recommendation from Kostiakov 1932 and Philip 1966). In this formulation, the infiltration function has two parts, where both the depth infiltrated and the infiltration rate match at the branch point

\[
D = k \tau^a \quad \text{and} \quad \tau = \frac{\Delta \theta}{K_e} \quad \text{for} \quad \tau \leq \tau_b
\]

\[
D = k \tau_b^a + b (\tau - \tau_b) \quad \text{and} \quad \tau = \tau_b \quad \text{for} \quad \tau \geq \tau_b
\]

which requires

\[
\tau_b = \left( \frac{ak}{b} \right)^\frac{1}{a-1}
\]

where \( \tau_b \) = time when the function branches. Note that while each parameter has the same units regardless of which equation is being used, for a given infiltration relationship the actual parameter values differ, depending on the equation chosen (see example in Streikoff et al. 2009a). With manipulation of terms, the second formula of Eq. (7) is the same as Eq. (2) with \( K_e = b \).

Two other ways of describing infiltration characteristics in surface irrigation are time-rated intake families as described by Merriam and Clemmens (1985) and the Soil Conservation Service (SCS) intake families (USDA 1974). The families have some practical advantages in terms of estimating the characteristics, especially in the planning stage of irrigation layout. The SCS families use
with \( c = 7 \) mm. When fit with a simple power function, all these families had relatively high exponents. The time-rated families were an empirical approach to overcoming this weakness in the SCS families. They relate the time to infiltrate 100 mm (in hours) \( \tau_{100} \) to the exponent \( a \), with the following empirical relationship

\[
a = 0.675 - 0.2125 \log_{10}(\tau_{100})
\]

They do not apply to cracking soils. A similar relationship can be developed for the SCS intake families. These relationships are described in more detail in Strelkoff et al. (2009b).

Horton (1940) proposed the following empirical equation

\[
D = k(1 - e^{-bT}) + bT
\]

where \( k, b \), and \( b \) = empirical constants. While the equation approaches a final infiltration rate, its use at short infiltration times is problematic (Clemmens 1981; Philip 1957), as will be discussed below.

### Infiltration Function Comparison

Philip (1966) and Hartley (1992) pointed out that the Modified Kostiakov and Philip equation have the same form. Thus, one might be able to physically interpret the Modified Kostiakov parameter, as long as \( a = 1/2 \), which they strongly recommended.

Ahuja et al. (2007) solved the Green-Ampt equation for different soils using soil physical properties reported by Rawls et al. (1982). The discussion above suggests that Eq. (5) is applicable to fit initial infiltration. Ahuja et al. (2007) fit their data to Eq. (7).

The resulting values of \( a \) for the initial infiltration ranged from 0.51 to 0.58, which are above the theoretical limit implied by the Philip equation of 1/2. They also demonstrate that the infiltration in 1 h, or the parameter \( K_r \), depends upon the initial water content and sorptivity (and hence \( K_d \)). The value increases a little with \( K_r \), whereas \( k \) increases greatly with \( K_r \). Had they fit Eq. (5) to the entire infiltration curve, they would have gotten larger values of \( a \), however the range of data generated was more representative of rainfall rather than irrigation (i.e., fit for a certain time period rather than for a representative irrigation depth). From their graphs for initial pressure head condition of \(-1,500\) kPa, we found the slope of the cumulative infiltration curve at 100 mm. The values of \( a \) were 0.715 and 0.699, versus 0.584 and 0.578 for initial infiltration, respectively. The other graphs were not plotted long enough to find similar values. The results are shown in Fig. 1, where the depth infiltrated at 1 h (\( k \) when expressed as mm/h) is shown as a function of \( a \). The initial infiltration curves of Ahuja et al. (2007) are shown as filled circles. The slope of the curves at 100-mm depth are shown as open circles. Also shown on the graph are SCS families when fit to a power function at 50 and 100 mm [i.e., to remove \( c \) in Eq. (8)]. Note that the theoretical values of \( a \) from Ahuja et al. (2007) at 100 mm are fairly close to the SCS families. This is not surprising since the original families were derived from the Philip equation (Walker et al. 2006).

In contrast, power law infiltration functions fitted from field measurements tend to have a \( <1/2 \). Fig. 1 also shows the Merriam and Clemmens (1985) time-rated families, along with the data from which they were generated. A large number of the points show values of \( a \) that are much smaller than \( 1/2 \). The time-rated families were developed specifically to address this difference in \( a \) values. Cracking soils were removed from the data shown in Fig. 1, so this difference cannot be explained simply from cracking. These data were mostly from research studies where experimental error could not explain these low \( a \) values.

Clemmens (1981) provided results from border irrigation evaluations that included ring infiltrometer data fit to Eq. (5) and adjusted to provide a volume balance for the irrigation event. Values of \( a \) ranged from 0.35 to 0.61. Bautista and Wallender (1993) identified infiltration parameters from furrow advance. Values of \( a \) ranged from 0.19 to 0.69. Manning (1993) determined infiltration functions for level furrows with a volume balance when surface volumes were measured over time. Values of \( a \) ranged from 0.21 to 0.53. A variety of other sources of data confirm this same pattern. When fit to a Kostiakov equation (straight line on log-log paper), observed values of the exponent \( a \) from surface irrigated fields are much smaller than is predicted by soil infiltration theory.

Results for \( a \) from a variety of studies where infiltration was determined from field data collected during the evaluation of irrigation events on a variety of soils are shown in Fig. 2. There are several data point not shown between 200 and 300 mm at \( a \) values between 0.2 and 0.3. The small triangles in Fig. 2 are results determined from field data collected during the evaluation of irrigation events on a variety of soils. The data were mostly from research studies where experimental error could not explain these low \( a \) values.

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representing preliminary results for Eq. (5), not the results reported. The data was collected on cracking soils in Egypt. Note that these values plot far from the circles for the time-rated families. However, the data from Manning (1993) show a similar pattern, but from a noncracking soil. Both data sets used field measured surface water depths to determine surface volume over time, from which subsurface volume could be determined. Thus they represent reasonable bay or furrow averaged infiltration.

Two things should be clear from Figs. 1 and 2. First, the observed field data are very different from the theoretical results. Even introducing soil layering into the Green-Ampt model would not result in fitted values of $a$ much below 0.5. Second, there is no general relationship between the magnitude of infiltration (as expressed by $k$) and the shape of the curve (as expressed by $a$). Consider the data point provided from Clemmens et al. (2003), the large open circle at depth infiltrated=39 mm and $a=0.37$. This was the result of very extensive measurement of surface volume over time with accurate measurements of inflow, advance, recession, etc., from which surface water profiles were accurately predicted with an unsteady-flow model and the computed infiltration function. Thus measurement accuracy does not cause this low value ($<1/2$) of $a$. Yet this data is for a sandy loam soil! There was no soil cracking, and no initial large infiltration amount. No tillage had occurred between the observed irrigation and the preceding one. Clearly, there has to be some changes in the soil structure or movement of soil particles to make such a dramatic change in infiltration from theory.

For irrigation practitioners, the issue is that they have to provide management recommendations that will provide improvements in performance. They cannot make such recommendations if they do not have a reasonable estimate of soil infiltration conditions. Our view is that theoretical infiltration equations, by themselves, do not provide representative soil infiltration estimates for surface irrigation management. Even use of the SCS families has led to difficulties with management, which led to the time-rated families. But even they are often not adequate.

**Adjustments to Theoretical Infiltration**

**Green-Ampt Time Adjustment**

Strelkoff et al. (2009b) discussed the wide variety of methods available to estimate the coefficients in the empirical infiltration equations. The idea behind the methods is to adjust these parameters so that the infiltration relationship matches the observations (e.g., advance, total infiltrated volume, etc.). Mailhol (2003) related soil physical parameters to terms in the Horton infiltration equation, and then used field observations to adjust those physical parameters, plus an additional empirical parameter. (To our knowledge this is the only published work that has attempted this.) For cracking soils, infiltration evolves toward a linear equation, which represents the asymptotic behavior of the Horton equation at long time. This is analogous to the second equation in the branch infiltration function [Eq. (7), where first term in second equation is a constant], or Eq. (2) from Philip (1966). The adjustment causes the calculated infiltration function to go to a final infiltration rate more quickly, with no change in the infiltrated depth or infiltration rate at long times. While the method has some merit, it does not appear to be sufficiently general. First, having no change in total depth infiltrated, regardless of the severity of cracking, goes against field observations, which suggest that much more water enters the soils over a given time when the crack are larger. Second, the method exhibits this behavior for relatively coarse textured soils, which frequently do not reach a final infiltration rate. Finally, even some cracking soils do not exhibit final-infiltration-rate behavior during an irrigation event, such as the Nile Delta soils in Egypt (El-Haddad et al. 2001).

The predictive form of the Horton equation used by Mailhol (2003), in the current notation, uses Eq. (10) where $k=0.95S^2/2K_r$, $b=K_r$, $\beta=\chi 2K_r^2/S^2$, and $S= -h_{t}a\Delta \theta$. He observed irrigation advance data and fit the volume infiltrated during advance (about 1 1/2 h). The following soil data were used: saturated hydraulic conductivity, $K_r=14.0$ mm/h [$K_r=(1/2)K$]; $h_t=-100$ mm, $\Delta \theta=0.21$. He obtained a good fit for advance with $\chi=15$. Note that this “fit” during advance essentially provides the correct average infiltration depth from 0 to 1 1/2 h, or the correct area under the infiltrated depth curve up to 1 1/2 h. (Actual average depth deviates a bit from this value when advance is nonlinear.) Mailhol (2003) provides only the general approach used to determine this average depth, without details. Fig. 3 shows the results for the Mailhol (2003) Horton equation solution and solution of the Green-Ampt equation [Eq. (2)]. Both used the same parameter values. For the Green-Ampt equation, the depth of water on the surface $h_t=50$ mm. The solutions for different values of $\chi$ are shown in Fig. 3 (with larger values of $\chi$ to the upper left). A higher value causes the solution to go to the final infiltration rate sooner. Note that at long times (e.g., $>4$ h), the solutions are identical. The solution for the field data reported by Mailhol (2003) was $\chi=15$, which gave a nearly constant infiltration rate after about 1/2 h, while the Green-Ampt solution did not reach a constant rate, even after 8 h. This was a loam soil and corresponded closely to the parameters for the loam soil in Ahuja et al. (2007). The long-term behavior of the Horton equation does not appear to be reasonable for this soil. Mailhol (2003) reported strong confidence on the hydraulic parameters based on his work and work reported by others at that site. The hydraulic conductively for the Green-Ampt equation, with all other terms the same, would have to be about 30% greater to match observed infiltration during advance. No data are provided from which to predict infiltration beyond the advance time of roughly 1 1/2 h.

One issue is that soil physical models assume a homogeneous, continuous soil mass that is not changing in density. What we observe in the field are significant changes in density as the soil is wetted. One approach would be to measure all these density changes and crack closures over time. This is not practical for routine application in evaluation of irrigation events. So the intent...
here is to find a balance between theory and empiricism. Mailhol (2003) found an innovative way to adjust the Horton equation to match field data, but unfortunately the method lacks general applicability. A parallel method for the Green-Ampt infiltration equation is not obvious.

Consider the influence of initial lower bulk densities and of soil cracks on infiltration. Water moves into these soils much more quickly than it would had the soil been at its final bulk density (without cracks) at the end of the irrigation. With surface irrigation the amount of water that moves into cracks before they close can be a substantial amount, for example, 50 mm or more. However, the soils tend to consolidate and the cracks close relatively quickly after wetting, say within a few minutes to half an hour, typically. After that time, one might expect infiltration to follow soil physical models more closely, since the changes in bulk density and cracking have now stopped. We suggest that the amount of water that has infiltrated already will influence infiltration in a similar manner from this time on, regardless of how long it took to infiltrate. In this sense, the cracking or low bulk density causes the infiltration time to be foreshortened. This is consistent with the approach of Bouwer (1969) for computing infiltration in layered soils (i.e., time offset). This would be equivalent to computing the infiltration with Eq. (4), with \( L_1 \) (and thus infiltrated, \( D_1 = \Delta \theta L_1 \) and \( \tau_1 \) (the time to infiltrated \( D_1 \)) determined empirically.

Fig. 4 shows the Horton curve estimated based on the observed data, with the Green-Ampt equation using the same parameter, but shifted 0.39 h to the left. This new Green-Ampt curve gives the same average infiltrated depth between 0.0 and 1.5 h. As one can see, this shift in time can be used to alter the infiltrated depth-time relationship. The shifted Green-Ampt curve is probably only valid after some (short) time where the cracks close or the bulk density stabilizes. With cracks, one might expect sudden infiltration as water flows into the cracks. However, other gradual changes in soil density would likely cause the infiltration to be a little less sudden, and approach the theoretical infiltration curve a bit more gradually; although perhaps still more suddenly than predicted by the Horton equation. When the infiltration curve Mailhol (2003) is fit to the second of Philip’s equations [Eq. (2)], \( G=19 \) mm and \( K_s=14 \) mm/h. When fit to the branch function \( b=14 \) mm/h, \( k=25 \) mm/h, \( a=0.3 \), and \( \tau_s=0.41 \) h.

It is recognized that the suggested adjustment procedure needs significant refinement and some rationale for how much adjustment to make in time (or \( L_1 \) and \( \tau_1 \)) versus other physical parameters. Note, that by increasing the hydraulic conductivity and reducing the time shift, one could produce an infiltration curve more similar to the Horton equations presented, and still match the average infiltrated depth during advance. It is also recognized that the 2D effects of furrow wetted perimeter are not taken into account. This will be the subject of future research.

**Green-Ampt Parameter Value Adjustments**

Data from an irrigation event was collected on a border strip (Field 6) within a sandy loam field near Wellton, Arizona on May 26, 1977. Standard irrigation data included inflow rate, advance, recession, and ring infiltrometers. The border strips were 186-m long and 53-m wide with a slope of 0.00033 and blocked to eliminate runoff. The flow rate was roughly 0.55 m/s. The average depth applied was 135 mm. The average infiltration opportunity time was 2.75 h. The crop was alfalfa. Advance and recession curves are given in Clemmens (1979).

In addition, soil samples were taken to determine; initial soil water (roughly 0.03), water content at saturation (0.27), soil texture (97% sand, 3% silt), and saturated hydraulic conductivity from a constant head permeability test (\( K_s=130 \) mm/h). An air-entry permeameter (Bouwer 1966) was used to determine the air-entry pressure, \( h_f \) (average value = -150 mm of water).

Eq. (3) was solved with \( K_s=65 \) mm/h \([1/2K_s] \), \( \Delta \theta=0.27 \) 0.03 = 0.24, \( h_f=-150 \) mm, and a water depth of 50 mm. The results are shown in Fig. 5, along with the ring data collected. First, note that there is considerable scatter in the ring data. However, most rings have a similar trend. For this irrigation, Clemmens (1979) reported \( k=73.7 \) mm/h and \( a=0.6 \) based on a postirrigation volume balance. This is shown by the heavy dashed line in Fig. 5 with a large dot indicating the average infiltration depth and average opportunity time, approximately representing the calibration point for the postirrigation volume balance. The Green-Ampt results are shown by the dotted line near the top of the data scatter. Both the ring data and the soil samples represent point samples, while the adjusted curve from estimation is a “field average.” The difference likely reflects the tendency to place rings or take soil samples where the crop is less dense, and thus perhaps more sandy.

The input parameters to the Green-Ampt model were adjusted to satisfy the postirrigation volume balance, as shown in Fig. 5 (Green-Ampt Adj). The adjusted parameters were \( K_s=35 \) mm/h
([1/2] \Delta \theta), \Delta \theta=0.15, and \Delta t=\alpha h_i=\alpha \frac{80}{\delta} \text{mm}. Note that the original Green-Ampt curve very closely matches the location and curvature of Ring 6, while the adjusted Green-Ampt curve matches very closely the location and curvature of Ring 4. (In hind sight, the value of \alpha appears to have been a composite taken from rings in Field 4 and Field 6.) This all suggests that the adjusted Green-Ampt infiltration curve represents the average field conditions. The adjusted Green-Ampt curve can be approximately fit to the Green-Ampt infiltration curve represents the average field conditions. The adjusted Green-Ampt curve can be approximately fit to the

\begin{equation}
A=29 \text{mm/h. With the recommended Manning } n=0.15 \text{ for alfalfa, simulation with this equation in the surface irrigation simulation program SRFR (Strelkoff et al. 2000) resulted in an advance time of 50 min, which matched the actual advance time, and was a better fit than the 54 min advance time (Clemmings 1979) predicted with the previously estimated equation (i.e., with } \alpha=0.6) \text{.}
\end{equation}

For this example with a sandy soil, a good fit to field infiltration data was achieved by simply changing the soil physical parameters, under the assumption that the point measured hydraulic properties were not representative of average field conditions. Adjusting for cracking was not appropriate since we had to reduce the amount of infiltration. There was no rationale for adjusting these parameters, except to fit the postirrigation volume balance. These values are all roughly half of the values found from experiments. From soils data, one would not expect these parameters to all change in the same direction by the same relative amount. Therefore, a more suitable strategy is needed for how to adjust these soil physical parameters. This is the subject of ongoing research.

Empirical Infiltration Equation Estimation

Most surface irrigation models do not have links to soil physical models for infiltration, although a few attempts have been made (e.g., Skonnard 2001, Zerihun et al. 2005). So, while such model efforts are under development, it is useful to see what the above results suggest regarding estimation with empirical equations.

From a practical standpoint, the instantaneous 12-mm infiltration shown in Fig. 4, caused by the time shift in the Green-Ampt equation, would not be a problem for either evaluation or simulation. The SRFR simulation model (Strelkoff et al. 2000) allows infiltration in the form

\begin{equation}
D = c + k t^a + b t^c \tag{11}
\end{equation}

The actual time for this “jump” in infiltration (c term) is short relative to advance times and relative to infiltration opportunity times. In fact several researchers have suggested a constant c, as in the SCS infiltration equation, except with a far larger value. Mailhol and Gonzalez (1993) suggested infiltration (for cracking soils) of the form [similar to Eq. (2)]

\begin{equation}
D = c + b t \tag{12}
\end{equation}

El-Haddad et al. (2001) found it convenient to fit infiltration with an equation of the form

\begin{equation}
D = c + k t^{1/2} \tag{13}
\end{equation}

They argued that with data fit to this equation, it was easier to understand changes from set to set or from irrigation to irrigation. The data could as easily have been fit to Eq. (5), but it was harder to interpret. Again, this was for a cracking soil that did not appear to exhibit a constant final infiltration rate during the time frame for irrigation events.

\begin{equation}
Q_{in} = \sigma_f A_{x} + \sigma_s A_{x} \tag{14}
\end{equation}

where \( t \) = time to advance to distance \( x \); \( Q_{in} \) = inflow rate (assumed constant in this equation); \( \sigma_f \) = surface shape factor (ratio of the average to the upstream surface cross sectional area); \( A_f \) = cross sectional flow area at the upstream end (volume per unit length or depth-times-width for border strips or basins); \( \sigma_s \) = subsurface shape factor (ratio of average infiltrated volume per unit length to the infiltrated volume per unit length at the upstream end); \( A_s \) = infiltrated area at the upstream end (volume per unit length, or depth times width for border strips or basins); and \( x \) is the advance distance at time \( t \). If we know the changes in inflow, outflow and surface volume over time, we also know the infiltrated volume over time. Integration of the infiltration function over time and distance should give the correct infiltrated volume over time. The two-point method solves this equation at advance to the field length and to half the field length. Further details and examples are given in Walker and Skogerboe (1987).

The usual approach is to use Eq. (6) for infiltration and to determine \( k \) and \( a \) from solution of the two volume balance equa-
tions, with \( b \) either known or obtained from inflow and outflow rates. Bautista et al. (2009) and Clemmens (2009) demonstrated that the two-point method can be very sensitive to the field conditions and the quality of the data, and can produce unreasonable values of \( a \). Further, in order for the infiltration function obtained to be useful, it must be extrapolated from the time of advance to the time for the full irrigation. Inappropriate estimates for the parameter \( a \) can cause this extrapolation to be very poor. In effect, Eqs. (5) and (6) are using the parameter \( a \) to account for initial infiltration conditions that deviate from theory. The proposed adjustment procedure for the Green-Ampt equation to account for these effects would be more consistent with the use of the parameter \( c \) to account for these effects. This suggests an infiltration equation of the form

\[
D = c + k\tau^{1/2} + bt
\]  

(15)

The equation reflects a balance between the more theoretical Philip equation and the need to match actual field conditions. It is possible to apply Eq. (15) to the two-point advance method. The two-point method generally assume a power function advance curve of the form

\[
t_k = s\tau^b
\]  

(16)

where \( s \) and \( h \) are empirical constants. Because each term in Eq. (15) has a different form of shape factor, it is easier to express each term separately. Assuming a constant infiltrated width \( W \) and infiltration from Eq. (15), Eq. (14) for advance to two advance locations \( x_1 \) and \( x_2 \) (usually \( 1/2 \) the field length and field length, respectively) with advance times \( t_1 \) and \( t_2 \) then becomes

\[
Q_Wt_1 = \sigma_Ax_1 + cWx_1 + \sigma_1k\tau_1^{1/2}Wx_1 + \sigma_2bt_1Wx_1
\]  

(17a)

\[
Q_Wt_2 = \sigma_Ax_2 + cWx_2 + \sigma_1k\tau_2^{1/2}Wx_2 + \sigma_2bt_2Wx_2
\]  

(17b)

For power function advance, the second subsurface shape factor is \( \sigma_2 = h/(h+1) \). With \( a = 1/2 \), the first shape factor simplifies to

\[
\sigma_1 = \frac{h + 1/3}{h + 1}
\]  

(18)

With the advance curve known, \( h \) is known and these two equations have three unknowns \( (k, c, \text{ and } b) \), rather than \( k, a, \text{ and } b \) in the standard two-point method. The proposed method here is to assume a value for one of these parameters and solve for the remaining two. A rational approach is proposed here for determining which of these parameters to fit. For advance to field end \( x_2 \), set \( b = 0 \) and \( c = 0 \), and solve Eq. (17b) for \( k \). Now, enter the resulting values for \( k \) into Eq. (17a) (still with \( b = 0 \) and \( c = 0 \)) and determine the advance time \( t_1 \) to half of the field length. If the actual and predicted advance times match, then a solution is found. If the computed advance time is less than the measured time, then initial infiltration is insufficient and we have to increase \( b \). If the computed advance time is more than the measured time, initial infiltration is too large and we have to reduce \( k \) by increasing \( b \). Once this decision is made, solve Eqs. (17a) and (17b) for either \( k \) and \( c \), or \( k \) and \( b \). The procedure can be explained with the advance curves shown in Fig. 7 which shows advance for a sloping border strip with a slope of 0.002, Manning \( n = 0.04 \), inflow rate=3 lps/m, \( a = 0.5 \), and \( k = 50 \text{ mm/h}^2 \). The exponent \( a \) was changed to 0.3 and \( k \) was adjusted to give the same advance time to the end of the field. This was repeated for \( a = 0.7 \). Note that when \( a = 0.3 \), advance is slower to the midpoint, resulting from more infiltrated volume. Adding the \( c \) term is similar to using a smaller \( a \) value and would increase the initial infiltrated volume.

When \( a = 0.7 \), advance is faster initially, resulting in less initial infiltrated volume. Adding the \( b \)-term is similar to using a larger \( a \) value and will decrease \( k \) and result in less initial infiltration.

It is recognized, that it may be appropriate in some cases for both \( b \) and \( c \) to be nonzero. In such cases, a separate estimate of \( b \) based on information later in the irrigation event is appropriate, with \( c \) and \( k \) determined from advance. The same approach above (but with \( b \) initially nonzero) would determine whether to increase \( c \) from 0 or to increase \( b \) from its initial value.

### Modified (Philip-Based) Two-Point Method Examples

For this example, one of the data sets of Elliot ([1980]. “Furrow irrigation field evaluation data. Summers of 1977–1979.” Internal Unpublished Report, Dept. of Agric. And Chem. Eng. Colorado State Univ., Fort Collins, Colo.] is used, as reported by Bautista et al. (2009). The standard two-point method is applied to the Benson 2.2-1 data set \([Q=1.14 \text{ l/s}, n = 0.02, x_2 = 625 \text{ m, } x_1 = 300 \text{ m, } t_0 = 513 \text{ min, } t_1 = 159 \text{ min, } \sigma_a = 0.75, \text{ bottom slope } \sigma_a = 0.00 439, \text{ and a trapezoidal furrow with } 0.2-\text{m bottom width and } 2:1 \text{ side slopes } (H:V)\). With \( b = 0 \) and \( c = 0 \), the standard method results in \( k = 18.39 \text{ mm/h}^2 \) and \( a = 0.409 \). Solving Eq. (17b) with \( a = 0.5, b = 0, \text{ and } c = 0; \) we get \( k = 15.79 \text{ mm/h}^2 \). With these values, Eq. (17a) gives \( t_1 = 134 \text{ min} \). Since the computed advance time is less than the actual advance time, we need to add \( c \) as an unknown. Solving Eqs. (17a) and (17b) with \( a = 0.5 \) and \( b = 0, \text{ we get } k = 3.54 \text{ mm/h}^2 \) and \( c = 4.9 \text{ mm} \).

Bautista et al. (2009) used a final infiltration rate equal to one half of the difference between inflow and outflow rates because outflow rates had not yet stabilized. For this case, \( b = 1.43 \text{ mm/h} \). Solving the standard two-point method then produced \( k = 17.22 \text{ mm/h}^2 \) and \( a = 0.297 \) (with \( b = 1.43 \text{ mm/h} \) and \( c = 0 \)). The modified procedure can also be solved with a known value for \( b \). This value is simply included in the calculations. The initial solution from Eq. (17b) with \( a = 0.5, b = 1.43 \text{ mm/h} \), and \( c = 0 \) resulted in \( k = 23.3 \text{ mm/h}^2 \). Solving Eq. (17a) then gives \( t_1 = 109 \text{ min} \), so \( c \) has to be increased even more when \( b = 0 \). Solving Eqs. (17a) and (17b) with \( a = 0.5 \) and \( b = 1.43 \text{ mm/h} \) gave \( k = 8.15 \text{ mm/h}^2 \) and \( c = 9.05 \text{ mm} \).
Summary and Conclusions

The Green-Ampt infiltration equation can be adjusted to match field conditions for coarse textured soils by adjusting the physical parameters. However, many soils exhibit significantly greater cumulative infiltration at short times than would be predicted from theory, such that adjusting the soil physical parameters is not a reasonable method to adjust cumulative infiltration. Foreshortening the initial infiltration time is proposed as a method for adjusting the Green-Ampt curve to match observed infiltration conditions. More detailed procedures need to be worked out to make both adjustments to the physical parameters and to the foreshortened infiltration time.

Extending these concepts to empirical infiltration equations suggest the use of a modified form of the Philip equation where a constant depth is added to account for cracking and consolidation. This is the same as the Modified Kostiakov equation with $a = 1/2$. This modified Philip equation is shown to be able to fit infiltration conditions as well as the Modified Kostiakov equation.

A modified two-point method is also presented for the modified Philip equation, which provides two options depending on the conditions encountered. One can solve for the parameters $k$ and $c$ or $k$ and $b$. This avoids problems reported with the two-point method which solves for $k$ and $a$.

References


