DYNAMIC RESPONSE OF AUTOMATIC WATER-LEVEL CONTROLLER. II: APPLICATION

By Albert J. Clemmens,1 Member, ASCE, Changde Wang,2 and John A. Replogle,3 Member, ASCE

ABSTRACT: Constant water-level controllers of various kinds are used within many irrigation canal distribution systems. In a companion paper, simple transfer-function equations were developed for one style of automatic water-level controller, the dual acting controlled leak system. Such information can be used to study the dynamic response of proposed designs. In this paper, these transfer-function equations are used to evaluate the stability and dynamic response of the system. It is shown that these systems are always stable. Response times and control decrement are well modeled.

INTRODUCTION

Automatic water-level controllers are useful tools within canal distribution systems. Most water-level controllers function to an approximate level (Replogle and Clemmens 1987). Because of the response of the canal system to changes in gate settings and flows, precise control is difficult. In the past, analytical evaluation of such controllers has been limited to static, steady-state conditions. There is a need to study the response of these controllers under dynamic, nonsteady conditions. While there are various ways to evaluate dynamic response, we chose to use linear systems analysis. This has the advantage that stability, damping ratios, etc., can be determined. It has the disadvantage of requiring the system to be described by a set of linear (differential) equations.

The dual-acting controlled leak system (DACL) is a hydraulic-mechanical device that requires neither electricity nor electronics (Clemmens and Replogle 1987a). It can be used to control upstream water levels, downstream water levels, or, when properly configured, off-take canal discharges. In many cases, it can be powered by the available energy loss across the controlled structure. When used for maintaining constant downstream flow rates on typical small irrigation canals, the downstream water level can be controlled to within about ±3 mm. There are basically two parts to the DACL system—the two leakage control valves and the actuating mechanism (see Fig. 1 in companion paper). The control valves regulate the leakage rate within the control system. A mismatch between supply and drain leakage rates results in a change in the actuating mechanism. The actuating mechanism for the DACL controller can be a radial gate, butterfly valve, inflatable bladder, or shutter valve in combination with a float and counterweight.

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Note. Discussion open until May 1, 1991. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on July 14, 1988. This paper is part of the Journal of Irrigation and Drainage Engineering, Vol. 116, No. 6, November/December, 1990. ©ASCE. ISSN 0733-9437/90/0006-0784/S1.00 + S.15 per page. Paper No. 25348.
(Replogle and Clemmens 1986; Clemmens and Replogle 1987b). The system has been studied in a hydraulics laboratory to determine the relevant factors affecting control. A field test was conducted on a typical radial gate (1.9-m radius) to study the system’s reliability in actual field operation. However, a number of unanswered questions remain on the sizing of the various components to produce the desired system response. Additional details on DACL operation and response are given in the companion paper.

In the companion paper, linear systems analysis (classical control theory) was used to develop transfer-function relations for the response of the DACL gate. In this paper, these transfer-function relations are used to evaluate the response of a field installation of a DACL gate. Stability, response, or setting time, and the control decrement will be determined from theory and compared with field results. In addition, the effects of hysteresis of the control values and pure delays on the control decrement will be examined. Finally, some recommendations and design limitations will be discussed.

**Feedback-Control Model**

In this paper, we examine only the transient response of the DACL control system for downstream flow-rate control without channel transient response. The DACL control system is assumed to be at the head measurement station of a critical-depth flume a short distance from the downstream side of the gate. The transient flow in this short section is approximated. See the companion paper for further details.

The feedback-control representation of the DACL controller was broken into three functional parts, or subsystems (as shown in Fig. 2 of the companion paper), namely, the input subsystem, the sensing subsystem, and the control subsystem. The input subsystem refers to the interaction between the water level immediately downstream from the gate, the downstream level at a flow-measuring structure a short distance downstream, the level upstream from the controlled gate, the gate opening, and the rate of flow through the gate. The sensing subsystem refers to the operation of the two leakage control valves in response to changes in downstream water level and to operating pressure. The control subsystem refers to the response of the actuating mechanism (gate, gate float, and counterweight) to changes in float-chamber water level.

The transfer-function block diagram for the DACL system is presented in Fig. 1. The equations for each block in Fig. 1 are given in Table 1. The details on the development of these equations are given in the companion paper. The variables in Table 1 are defined in Appendix II. Notation.

The following assumptions, which could affect conclusions drawn, were made in the development of these relations.

1. An exponential function is used to describe the change in water level at the flume a distance downstream from the gate caused by a change in gate opening.

2. The hysteresis is gate float, and gate movement is approximated with a linear function over the range of gate travel.

The remaining assumptions simply define the problem studied. Neither of these assumptions is significant for the problem studied here, but could be
significant for other applications of this device. With these assumptions, the predicted response time of the system to large changes in levels may be significantly in error. Response times for small changes should be reasonably well predicted, as should stability relations.

**Dynamic Response**

**Block Diagram Reduction**

The functioning of the DACL system is affected by two disturbances, as shown in Fig. 1. The first disturbance occurs with a change in upstream water level, $D_u$, and the second occurs with a change in the control setting,
### TABLE 2. Combined Transfer Functions for Block-Diagram Reduction

<table>
<thead>
<tr>
<th>Label (1)</th>
<th>Transfer-function relations (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_{30})</td>
<td>(\frac{T_2}{1 + T_2 T_3}) (\times) (\frac{1}{1 + T_2 T_3})</td>
</tr>
<tr>
<td>(T_{31})</td>
<td>(T_5 T_{31} + T_2 T_{32} + T_{1b})</td>
</tr>
<tr>
<td>(T_{32})</td>
<td>(\frac{T_2}{1 + T_2 (T_1 + T_{1b})}) (\times) (\frac{1}{1 + T_2 T_3})</td>
</tr>
<tr>
<td>(T_{33})</td>
<td>(\frac{T_2}{1 + T_2 (T_3 T_{35})}) (\times) (\frac{1}{1 + T_2 T_3})</td>
</tr>
<tr>
<td>(T_{34})</td>
<td>(T_3 + (T_1 T_{1b} + T_2 (T_2 T_{33} T_{35}))) (\times) (\frac{1}{1 + T_2 T_3})</td>
</tr>
<tr>
<td>(T_{35})</td>
<td>(\frac{T_2 T_3 T_{35}}{1 + T_2 T_3 T_{30} T_{35}}) (\times) (\frac{1}{1 + T_2 T_3})</td>
</tr>
<tr>
<td>(T_{36})</td>
<td>(\frac{T_2 T_3}{1 + T_2 T_3 T_{35}}) (\times) (\frac{T_{35}}{T_3} T_{35} T_{35} T_{35} T_{35})</td>
</tr>
<tr>
<td>(T_{41})</td>
<td>(\frac{T_2}{1 + T_{41}})</td>
</tr>
</tbody>
</table>

The dynamics of the system must be evaluated for both types of disturbances. Block-diagram reduction will be slightly different for these two disturbances, since the final block diagram should have the disturbance as the only input and the desired response as the only output. There are a number of possible variables that could be used as the desired output in evaluating stability: flume head, gate discharge, gate position, float-chamber water level, or float-chamber inflow rate. The relations for stability will not be affected by the output selected. The error in flume head will be used as the desired output, since this is required for evaluation of control accuracy.

The results of block-diagram reduction are given in Table 2. Methods for block-diagram reduction can be found in Groover (1980), as well as most textbooks on linear systems analysis. For a change in upstream water-level, disturbance \(D_a\), the response of the flume head, \(h_f\), is given by \(T_{35}\) (Table 2). For a change in the control setting, disturbance \(D_r\), the response of the flume head is given by \(T_{32}\) (Table 2). The output in terms of Laplace transform variable \(s\) is a simple algebraic equation relating inputs to output. However, the relation is complex enough such that a reverse Laplace transform is not easily found. (A reverse Laplace transform would result in a simple algebraic equation relating outputs to inputs with time as a variable instead of \(s\)).

#### Stability

The general strategy for determining stability is to expand the denominator of the final transfer function \((T_{35}\) or \(T_{32}\)) so that it can be expressed as a polynomial in \(s\). This polynomial set equal to zero is called the “characteristic equation.” The characteristic equation is then factored. If the roots of this equation are all negative, or, in the case of complex roots, if all the roots have negative real parts, then the system is stable (Johnson and Johnson 1975). The characteristic equation is of the form

\[
as^2 + bs + c = 0.\tag{1}\n\]

where \(a\), \(b\), and \(c\) = coefficients. Both relations \(T_{35}\) and \(T_{42}\) have the same characteristic equation, the coefficients of which are given in Table 3, where

\[
X = [\alpha A_n + (1 - \alpha)A_a]^{-1}.\tag{2}\n\]

For a quadratic expression, these stability criteria are satisfied only if all the coefficient values are positive (or all negative). All of the original trans-
fer functions of Table 1 are positive quantities by definition. The combined transfer functions of Table 2 contain only positive combinations of the original transfer functions, and thus are also all positive. The coefficients of the characteristic equations (Table 3) contain only positive terms and are all positive. Thus, stability is guaranteed under all conditions, within the limitations of assumptions made in this analysis.

**Final Value (Decrement)**

According to the Laplace-transform theorem, when the input is a step function, the limit of the output as time approaches infinity (final value) equals the limit of the Laplace-transformed output as \( s \) approaches zero (Johnson and Johnson 1975). The final value essentially gives the control decrement caused by a step change in input conditions.

**Change in Upstream Head**

The numerator for the Laplace-transformed output function is given in Table 4 and the denominator is Eq. 1 with the coefficients from Table 3. Setting \( s = 0 \) gives the expected final value of the system to a step change in upstream level

\[
\lim_{s \to 0} T(s) = \frac{(T_{17} + T_{18} + T_{12}T_{16})(T_{5}T_{30}T_{11} + T_{14}T_{16} + T_{17})(T_{31})^{-1}}{T_{17} + T_{18} + T_{12}T_{16} + T_{2}T_{7}T_{25}(T_{5}T_{11} + T_{8}T_{12} + T_{16})(1 + T_{2}T_{3})^{-1}}
\]  

(3)

The term \((T_{17} + T_{18} + T_{12}T_{16})\) represents the effect of the change in float-chamber level on leakage discharge. If this minor effect is ignored, Eq. 3 reduces to zero.

**Change in Control Setting**

The numerator and denominator of the transformed output function are found as Eq. 3

\[
\lim_{s \to 0} T(s) = \frac{T_{17} + T_{18} + T_{12}T_{16}}{T_{17} + T_{18} + T_{12}T_{16} + T_{2}T_{7}T_{18}T_{25}(1 + T_{2}T_{3})^{-1}}
\]

(4)

Again, the numerator contains the term related to changes in float-chamber level. Ignoring this, one can reduce Eq. 4 to zero.

**TABLE 4. Numerators for Final Transfer Functions**

<table>
<thead>
<tr>
<th>Label (1)</th>
<th>Coefficient equation (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{38} )</td>
<td>( [s + X(T_{17} + T_{18} + T_{12}T_{16})]<a href="T_%7B31%7D">T_{5}T_{30}T_{11} + (st_{1} + 1)T_{14}T_{16} + T_{17}</a>^{-1} )</td>
</tr>
<tr>
<td>( T_{42} )</td>
<td>( (t_{1}s + 1)[s + X(T_{17} + T_{18} + T_{12}T_{16})] )</td>
</tr>
</tbody>
</table>
Control Accuracy

When there is a delay in response of a control system, the output tends to oscillate around the final value (unless it is over-damped). The magnitude of this oscillation plus the decrement is referred to here as the control accuracy. In this analysis, we have ignored the pure delays associated with gate frictional resistance and channel travel time. The former should have little, if any, impact on the control accuracy. However, by linearizing this function, one can reduce the rate of gate movement significantly. The channel response lag will probably have a significant effect on control accuracy. Since we have not modeled this factor precisely, the model developed will not give reasonable values for control accuracy.

Settling Time

The settling time of the system is sometimes considered the time required for the output to come within 2% of the final value of the output, relative to the initial step input. For example, for a unit step input (1.0) and a final value of 0.5, the settling time would be the time to reach 0.52 (or 0.48 if approached from the bottom). This settling time can only be computed once the reverse Laplace transform of the system has been determined.

System Gain (Leverage)

For the situation described in this paper, the difference between minimum upstream level and maximum downstream level represents the range of water levels in the float chamber. If the head drop across the gate is small, the change in float-chamber water levels may be insufficient for full gate travel. To overcome this, increased leverage is needed between the float and the gate. The gain factor, $K_x$, represents the ratio between gate and float movement. This increased leverage can be applied by using different lever and gear systems (Replogle and Clemmens 1986). However, this increased leverage requires an increased buoyant force to overcome gate friction, and which, in turn, requires a larger float and float chamber. Thus, the settling time of the system may not be affected by gain, while the control decrement may be reduced by increasing gain.

Hysteresis

The effects of channel time lag have been partially accounted for by relation $T_{12}$ in Table 1. This relation smooths out the pure delay with an exponential function. A worst-case scenario would be represented by a rapidly moving gate, which would open too far during the time for the flow change to move to the sensing level downstream, resulting in an overcorrection. The amount of this over-correction relative to the error at a given time, $\delta h_c$, can be found from the following relation:

$$\delta h_c = (T_{11}T_{12} + T_{12}T_{21})T_2T_3T_4T_5T_6T_7T_8T_9T_{10}T_{11} \delta h_c$$

which represents the major feedback path with only the delay time $t_c$ (i.e., $t_c$ replacing $T_{21}$). Of course, if the control level is approached gradually, this error approaches zero.

The delay caused by gate friction is also partially accounted for in the linearized model, in that after the gate starts to move, the forces are properly balanced. A worst-case scenario can be determined by examining the effects of a pure change in the float-chamber level, $\delta H_c$, on the control error, $\delta h_c$. 789
Examining the direct influence of $\delta H_s$ on $\delta h_r$ gives, from Fig. 1

$$\delta h_r = T_{ts}(T_b)^{-1}H_r$$

(6)

where $\delta H_s$ is equated to the frictional hysteresis, $H_r$.

So far, we have not accounted for possible frictional resistance in the control valves. We could have hypothesized a linearized hysteresis, but chose not to do so in this paper. The effects of friction in the control valves have a direct impact on the control error, since they translate directly (through the float lever arm) to change in buoyant force. We hypothesized that the valve drag is directly proportional to the pressure on the plunger. The force on the end of the plunger is converted to a normal force on the valve-side wall. This normal force then acts with static friction to resist plunger movement. The exact relation between these forces depends on the position of the lever arms. Near the equilibrium position for these valves, a majority of this force is transferred to the walls. The error in control level for the supply valve is then related to this pressure by

$$\delta h_r = T_{ts}(T_b)^{-1}\mu h_u$$

(7)

where $\mu$ = static friction factor, which includes both the transfer of plunger pressure to normal force and the wall static friction. Eq. 7 applies to the drain valve when $T_b$ replaces $T_s$ and $H_s$ replaces $h_u$. Eq. 7 represents a potential “static” change in control level.

**MODEL VERIFICATION**

A full-scale installation of the DACL system was tested in 1985. The results of these field tests were published by Clemmens and Replogle (1987a) to demonstrate the basic functioning of the DACL system. Here we use the field data to verify the model presented in Fig. 1 and Tables 1–4. The test gate was a 2.4-m wide, 1.7-m high, 1.9-m radius, radial gate that had been installed at a field testing station by the Salt River Project, a local irrigation district (Phoenix, Ariz.). The conditions at the start of run I (Clemmens and Replogle 1987a) are given in Table 5. The values of the associated transfer functions are given in Table 6. For this setup, there was a considerable drop (1.3 m) between the upstream channel bottom at the gate and the flume crest.

<table>
<thead>
<tr>
<th>Variable (1)</th>
<th>Value (2)</th>
<th>Constant (3)</th>
<th>Value (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>0.536 m$^3$/s</td>
<td>$\mu$</td>
<td>2.15</td>
</tr>
<tr>
<td>$h_s = h_u$</td>
<td>0.342 m (0.0)</td>
<td>$K_{si}$</td>
<td>8.0</td>
</tr>
<tr>
<td>$h_u$</td>
<td>0.567 m (1.867)</td>
<td>$K_{sr}$</td>
<td>8.0</td>
</tr>
<tr>
<td>$S_r$</td>
<td>0.126 m</td>
<td>$A_p$</td>
<td>$5.07 \times 10^{-4}$ m$^2$</td>
</tr>
<tr>
<td>$q_t = q_s$</td>
<td>$5.00 \times 10^{-5}$ m$^3$/s</td>
<td>$A_d$</td>
<td>0.148 m$^2$</td>
</tr>
<tr>
<td>$S_{pp}$</td>
<td>0.002 m</td>
<td>$A_r$</td>
<td>1.169 m$^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>0.025 m</td>
<td>$r$</td>
<td>0.0485 m</td>
</tr>
<tr>
<td>$H_s$</td>
<td>1.220 m</td>
<td>$K_s$</td>
<td>1.0</td>
</tr>
<tr>
<td>$a$</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_c$</td>
<td>21.4 s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Label (1)</td>
<td>Transfer-function relations (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>---------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_2$</td>
<td>0.2968</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_3$</td>
<td>0.4691 (0.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_7$</td>
<td>4.254</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{11}$ = $T_0$</td>
<td>0.125</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{16}$</td>
<td>0.00140</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{17}$</td>
<td>$3.864 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{18}$</td>
<td>0 ($2.049 \times 10^{-3}$ m$^2$/s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{22}$</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{26}$</td>
<td>1.5186 s$^{-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{27}$</td>
<td>$(1 + 21.43)^{-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{33}$</td>
<td>0.2968</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{35}$</td>
<td>0.00625</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{36}$</td>
<td>1.5186 ($s + 9.378 \times 10^{-5}$)$^{-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{37}$</td>
<td>1.5186 ($s + 1.493 \times 10^{-5}$)$^{-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{38}$</td>
<td>0.5088 + 0.8495 s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{39}$</td>
<td>$[0.011782 s^2 + 0.0070584 s + 1.0535 \times 10^{-4}] / [(s + 0.03964)(s + 0.007240)]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{41}$</td>
<td>$(0.00592)/(21.4 s + 14 s + 0.0001493)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{42}$</td>
<td>$[-(s^2 + 0.046879 s + 6.9762 \times 10^{-4}] / [(s + 0.03964)(s + 0.007240)]$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(which was level with the local channel bottom), making the gate free flowing. Thus, the head, $h_u$, on the gate and the head, $h_v$, on the valve are different numbers, as indicated in Table 5. Also, there is no feedback of downstream level on gate discharge ($T_3$ as feedback to $T_3$ in Fig. 1 is zero). The outlet for the drainage drain was at a fixed level, open to the atmosphere (thus $T_{18} = 0.0$) for response to downstream level. $T_{18}$ is not zero when used for response to float-chamber level.

The final transfer functions ($T_{35}$ and $T_{22}$) must be divided by $s$ to obtain the transfer function for a step change in input, $r(s)$. Then the denominator must be broken up into separate terms by the method of partial fractions. The resulting transfer-function equation is

$$r(s) = \frac{A}{s} + \frac{B}{s + 0.03964} + \frac{C}{s + 0.007240}$$  \hspace{1cm} \hspace{1cm} (8)

The final system model is obtained by taking the reverse Laplace transform of Eq. 5, namely

$$r(t) = A + Be^{-0.03964r} + Ce^{-0.007240r}$$  \hspace{1cm} \hspace{1cm} (9)

The coefficient values for system response, in terms of error in flume head from desired, for a step change in the upstream level and for a step change in the control setting, are given in Table 7, and the response functions are plotted in Fig. 2. The value of $A$ represents the final value or decrement. The sum of the coefficient values represents the initial response of the system to an instantaneous step change. The settling time is found by finding the time in Eq. 6 when $r(t)$ reaches 2% of $A$ (2% relative to the change in level that would result without the control system in place). These times were 10.0 and 9.5 minutes, respectively. It should be remembered that these results represent the response of the system from one given position to an
TABLE 7. Coefficients for Final System Equations

<table>
<thead>
<tr>
<th>Coefficient (1)</th>
<th>Step Change in</th>
<th>Meaning (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upstream level</td>
<td>Control setting</td>
</tr>
<tr>
<td>A</td>
<td>0.00367</td>
<td>-0.02431</td>
</tr>
<tr>
<td>B</td>
<td>-0.20262</td>
<td>0.21801</td>
</tr>
<tr>
<td>C</td>
<td>0.21073</td>
<td>-1.19370</td>
</tr>
<tr>
<td>Sum</td>
<td>0.01178</td>
<td>-1.00000</td>
</tr>
</tbody>
</table>

-infinitesimal step change (i.e., the slope of the response function at a point). Because of assumptions made in this analysis, these results may not be exactly correct for large changes, as have been measured for verification.

For a step change in upstream level, the flume head would change by 14.0% of that step change without the control system in place and by only 0.28% with the control system in place. For example, a 1.0-m change in upstream head would result in a change in flume head of only 3 mm. Thus, the system as configured in these tests has a very small decrement. The decrement caused by changes in upstream water level was barely detectable in the field installation. In the laboratory experiments reported by Clemmens and Replogle (1987), a change of 0.67 m in upstream head caused roughly 3 mm of change in the control level. The response of the laboratory setup would be slightly different than for the field installation, but it shows the correct order of magnitude. The laboratory setup was not modeled because of difficulty in determining the discharge relation of the butterfly valve.

The step change in control level setting should cause the control level to change by approximately $100 - 1.8\% = 98.2\%$ of that step change. A 10-mm change in control setting would thus result in a 9.8-mm change in flume

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![Graph](image)

**FIG. 2. Response of Control Level Error to Unit Step Changes in Input**

792
head. In the field tests, a step change of \(-10\) mm was made (from 343 mm to 333 mm) and the system responded with a change in water level of \(-9.7\) mm in about 10 minutes. This would represent a good match to model results. However, because we did not have exact control over all conditions at the site, the upstream level rose 0.04 m. This caused the chamber level to continue to fill and the gate to continue to close for an additional 5 or 6 minutes. Thus, the response time is well predicted by the model. Unfortunately, the error in the change in level was not as well predicted. During this additional five or six minutes, the level continued to drop an additional 1.5 mm, giving a total change of \(-11.2\) mm. For this rise in upstream water level, the model would have predicted about a 0.1-mm increase in level, making the total predicted change \(-9.7\) mm. Water level readings fluctuated, and varied by \(\pm 0.5\) to \(\pm 1\) mm.

After the first chance in control level of 10 mm, the system was allowed to stabilize; then, the control level was returned to its original value. The response time again was about 8 min. Both the flume head and upstream level changed. This time, the flume level returned to 348.6 mm, while the upstream level dropped roughly 0.08 m. In this case, the model did not do a good job of predicting control accuracy.

**Hysteresis**

Solving Eq. 5 with the data from Tables 5 and 6 gives \(\delta h_{dp} = 0.14 \delta h_c\). Therefore, for a control error of 10 mm, the apparent decrement caused by response delay would be 1.4 mm. However, since the level is approached gradually (10-min settling time versus 20-sec travel time), the error would be much less. When the control error is 1 mm, the apparent decrement is only 0.14 mm, an insignificant amount. Thus, channel travel time would not have a significant impact on this system, since gate movement is so slow relative to channel response. Hysteresis caused by gate friction was also evaluated. Solving Eq. 6 with \(H_c = 35\) mm gives \(\delta h_c = 0.4\) mm, again a relatively insignificant amount.

Assuming that the static friction force of the control valve represented 0.1 times the total force on the valve plunger (i.e., \(\mu = 0.1\) in Eq. 7) resulted in values of control error of 2.1 and 1.4 mm for the supply and drain valves, respectively. Combining these two and considering the hysteresis in both directions would result in a control band of 7 mm, which is close to that observed in the field. This would suggest that the control valve friction is the major contributor to the control band observed in the laboratory and in the field. This also confirms field observations that when the system was at "equilibrium," "bouncing" the control valves resulted in a change in the equilibrium level. This was a result of changing the valve hysteresis!

Under test run 1, the system started out approximately at one side of the float-chamber hysteresis loop, and ended at the other side, as evidenced by a 100-mm difference in float-chamber water level. In addition, the control valves would also be on opposite sides of their hysteresis loop. This explains why the control level did not return to the same level, but was 6 mm higher, representing the impact of the hysteresis loop of the control valves.

**Comments**

While we originally thought (Clemmens and Replogle 1987a) that gate friction was necessary for the stable control of these systems, these results
demonstrate that this was not the case. These systems are stable regardless of the size or scale of any of their physical dimensions. All that will change will be the response time, decrements, and control band. A large float and float chamber will keep system response slow. Of high significance are the frictional properties of the control valves and whether they change over time. The less friction there is, the smaller the control band. If large changes in upstream head or source pressure are expected (as for a regulating reservoir), a constant upstream supply for the control valves is recommended.

This model can be used to test different combinations of component sizes and should aid in the design process. A major drawback is the need to compute characteristic equations for each set of design conditions. While a numerical simulation of the model presented would be useful for testing sample cases and would be a great asset for design, developing general results from it would be difficult.

Automatic gates are frequently used in canal systems where the level to be controlled is influenced by the dynamic response of the canal system. We have purposely avoided this problem, as this model is considered simply one component of the overall system. When such control gates are designed for use in a canal system, the impact of the structure on canal-system response should be evaluated.

CONCLUSIONS

The control mechanism of the DACL controller was examined by constructing a feedback-control model with linear systems analysis. The model constructed was second order. Hysteresis in the gate-float movement caused by gate friction was approximated with a linear relation. The canal response between the gate and control location was approximated with an exponential function, since the pure delay of the travel time could not be adequately modeled. The remaining portions of the feedback control systems were modeled with coefficients based on some initial conditions. Thus, an individual response equation actually represents the slope of the system response at that given point.

First, we showed that the system is always stable regardless of the physical dimensions chosen, and that response time could be adequately predicted with the model. The control decrement was adequately modeled; however, the control accuracy was not well modeled with the linear system. The control accuracy was well predicted by examining the hysteresis effects of the control valves.

The model was tested with data reported by Clemmens and Replogle (1987) for the system configured to maintain a constant discharge downstream from a radial off-take gate. The decrement in control for a change in upstream level was only 0.3%, while the settling time was roughly 10 min. For a change in control setting, the decrement was roughly 2.0% and the settling time was roughly 9.5 min. Gate friction and channel delays had an insignificant effect on control accuracy, but control valve friction proved to be the major contributor to the 6 to 7 mm control band.

APPENDIX I. REFERENCES


794

APPENDIX II. Notation

The following symbols are used in this paper:

$A$ = coefficient in response equation;
$A_d$ = annular area between float and float chamber walls;
$A_f$ = area of float chamber;
$A_p$ = area of valve plunger;
$a$ = coefficient in quadratic formula;
$B$ = coefficient in response equation;
$b$ = coefficient in quadratic formula;
$C$ = coefficient in response equation;
$c$ = coefficient in quadratic formula;
$D_c$ = change in set control level;
$D_u$ = disturbance or change in water level upstream from gate;
$e$ = base of natural logarithms;
$H_F$ = change in float-chamber water level required to overcome gate friction;
$H_f$ = water level in float chamber;
$h$ = control float submergence depth, water depth, or head;
$h_d$ = canal water depth immediately downstream from gate;
$h_f$ = head above flume crest;
$h_u$ = canal water depth immediately upstream from gate;
$K_g$ = leverage ratio between gate and gate float;
$K_r$ = leverage ratio between control valve plunger and control valve float;
$Q$ = canal flow rate;
$q$ = controlled leakage rate;
$r$ = control float radius;
$S_g$ = gate opening;
$S_p$ = control valve plunger position;
$s$ = Laplace transform variable;
$T(s)$ = Laplace transform representation;
$t$ = time;
$t_c$ = time for celerity wave to travel from gate to flume;
$u$ = exponent in flume head-discharge relation;
$X$ = defined function;
\( \alpha \) = slope of float response to change in float-chamber level;
\( \delta \) = incremental change in a variable;
\( \delta h_r \) = error in controlled level;
\( \delta h_{sf} \) = overcorrection in controlled level;
\( \pi \) = ratio of area to circumference of circle;
\( \mu \) = static friction factor.

Subscripts
\( d \) = downstream;
\( f \) = control floats (or referenced to flume crest);
\( g \) = gate;
\( i \) = inlet or supply leakage control valve;
\( o \) = outlet or drain leakage control valve; and
\( u \) = upstream.