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A Slug Test for Determining Hydraulic Conductivity of Unconfined Aquifers With Completely or Partially Penetrating Wells

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A procedure is presented for calculating the hydraulic conductivity of an aquifer near a well from the rate of rise of the water level in the well after a certain volume of water is suddenly removed. The calculation is based on the Thiem equation of steady state flow to a well. The effective radius R_e over which the head difference between the equilibrium water table in the aquifer and the water level in the well is dissipated was evaluated with a resistance network analog for a wide range of system geometries. An empirical equation relating R_e to the geometry of the well and aquifer was derived. The technique is applicable to completely or partially penetrating wells in unconfined aquifers. It can also be used for confined aquifers that receive water from the upper confining layer. The method's results are compatible with those obtained by other techniques for overlapping geometries.

With the slug test the hydraulic conductivity or transmissibility of an aquifer is determined from the rate of rise of the water level in a well after a certain volume or 'slug' of water is suddenly removed from the well. The slug test is simpler and quicker than the Theis pumping test because observation wells and pumping the well are not needed. With the slug test the portion of the aquifer 'sampled' for hydraulic conductivity is smaller than that for the pumping test even though with the latter, most of the head loss also occurs within a relatively small distance of the pumped well and the resulting transmissibility primarily reflects the aquifer conditions near the pumped well.

Essentially instantaneous lowering of the water level in a well can be achieved by quickly removing water with a bailer or by partially or completely submerging an object in the water, letting the water level reach equilibrium, and then quickly removing the object. If the aquifer is very permeable, the water level in the well may rise very rapidly. Such rapid rises can be measured with sensitive pressure transducers and fast-response strip chart recorders or x-y plotters. Also it may be possible to isolate portions of the perforated or screened section of the well with special packers for the slug test. This not only reduces the inflow and hence the rate of rise of the water level in the well, but it also makes it possible to determine the vertical distribution of the hydraulic conductivity. Special packer techniques may have to be developed to obtain a good seal, especially for rough casings or perforations. Effective sealing may be achieved with relatively long sections of inflatable stoppers or tubing. The use of long sections of these materials would also reduce leakage flow from the rest of the well to the isolated section between packers. This flow can occur through gravel envelopes or other permeable zones surrounding the casing. Sections of inflatable tubing may have to be long enough to block off the entire part of the well not used for the slug test. High inflation pressures should be used to minimize volume changes in the tubing due to changing water pressures in the isolated section when the head is lowered.

So far, solutions for the slug test have been developed only for completely penetrating wells in confined aquifers. *Cooper et al.* [1967] derived an equation for the rise or fall of the water level in a well after sudden lowering or raising, respectively. Their equation was based on nonsteady flow to a pumped, completely penetrating well, and the solution was expressed as a series of 'type curves' against which observed rates of water level rises were matched. Values for the transmissibility and storage coefficient were then evaluated from the curve parameter and horizontal-scale position of the type curve showing the best fit with the experimental data. *Skibitzke* [1958] developed an equation for calculating transmissibility from the recovery of the water level in a well that was repeatedly bailed. The technique is limited to wells in confined aquifers with sufficiently shallow water levels to permit short time intervals between bailing cycles [*Lohman*, 1972].

To use the slug test for partially penetrating or partially perforated wells in confined or unconfined aquifers, some solutions developed for the auger hole and piezometer techniques to measure soil hydraulic conductivity [*Bouwer and Jackson*, 1974] may be employed. However, the geometry of most groundwater wells is outside the range in geometry covered by the existing equations or tables for the auger hole or piezometer methods. For this reason, theory and equations are presented in this paper for slug tests on partially or completely penetrating wells in unconfined aquifers for a wide range of geometry conditions. The wells may be partially or completely perforated, screened, or otherwise open along their periphery. While the solutions are developed for unconfined aquifers, they may also be used for slug tests on wells in confined aquifers if water enters the aquifer from the upper confining layer through compression or leakage.

THEORY

Geometry and symbols of a well in an unconfined aquifer are shown in Figure 1. For the slug test the water level in the well is suddenly lowered, and the rate of rise of the water level is measured. The flow into the well at a particular value of y can be calculated by modifying the Thiem equation to

$$Q = 2\pi KL \frac{y}{\ln (R_e/r_w)} \tag{1}$$

where Q is the flow into the well (length^3/time), K is the hydraulic conductivity of the aquifer (length/time), L is the height of the portion of well through which water enters (height of screen or perforated zone or of uncased portion of well), y is the vertical distance between water level in well and equilibrium water table in aquifer, R_e is the effective radius over which y is dissipated, and r_w is the horizontal distance

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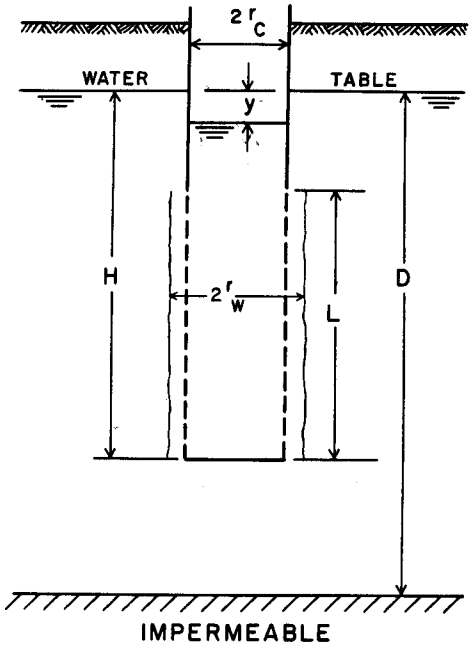

Fig. 1. Geometry and symbols of a partially penetrating, partially perforated well in unconfined aquifer with gravel pack or developed zone around perforated section.

from well center to original aquifer (well radius or radius of casing plus thickness of gravel envelope or developed zone).

The terms L, y, R_e, and r_w are all expressed in units of length. The effective radius R_e is the equivalent radial distance over which the head loss y is dissipated in the flow system. The value of R_e depends on the geometry of the flow system, and it was determined for different values of H, L, D, and r_w (Figure 1) with a resistance network analog, as will be discussed in the next section. Equation (1) is based on the assumptions that (1) drawdown of the water table around the well is negligible, (2) flow above the water table (in the capillary fringe) can be ignored, (3) head losses as water enters the well (well losses) are negligible, and (4) the aquifer is homogeneous and isotropic. These are the usual assumptions in the development of equations for pumped hole techniques [*Bouwer and Jackson*, 1974, and references therein].

The value of r_w in (1) represents the radial distance between the undisturbed aquifer and the well center. Thus r_w should include gravel envelopes or 'developed' zones if they are much more permeable than the aquifer itself (Figure 1).

The rate of rise, dy/dt, of the water level in the well after suddenly removing a slug of water can be related to the inflow Q by the equation

$$dy/dt = -Q/\pi r_c^2 \tag{2}$$

where πr_c^2 is the cross-sectional area of the well where the water level is rising. The minus sign in (2) is introduced because y decreases as t increases.

The term r_c is the inside radius of the casing if the water level is above the perforated or otherwise open portion of the well. If the water level is rising in the perforated section of the well, allowance should be made for the porosity outside the well casing if the hydraulic conductivity of the gravel envelope or developed zone is much higher than that of the aquifer. In that case the (open) porosity in the permeable zone must be included in the cross-sectional area of the well. For example, if the radius of the perforated casing is 20 cm and the casing is surrounded by a 10-cm permeable gravel envelope with a porosity of 30%, r_c should be taken as $[20^2 + 0.30(30^2 - 20^2)]^{1/2} = 23.5$ cm to obtain the cross-sectional area of the well that relates Q to dy/dt. The value of r_w for this well section is 30 cm.

Combining (1) and (2) yields

$$\frac{1}{y}\,dy = -\frac{2KL}{r_c^2 \ln (R_e/r_w)}\,dt \tag{3}$$

which can be integrated to

$$\ln y = -\frac{2KLt}{r_c^2 \ln (R_e/r_w)} + \text{constant} \tag{4}$$

Applying this equation between limits y_0 at $t = 0$ and y_t at t and solving for K yield

$$K = \frac{r_c^2 \ln (R_e/r_w)}{2L}\,\frac{1}{t}\ln\frac{y_0}{y_t} \tag{5}$$

This equation enables K to be calculated from the rise of the water level in the well after suddenly removing a slug of water from the well. Since K, r_c, r_w, R_e, and L in (5) are constants, $(1/t)\ln y_0/y_t$ must also be constant. Thus field data should yield a straight line when they are plotted as $\ln y_t$ versus t. The term $(1/t)\ln y_0/y_t$ in (5) is then obtained from the best-fitting straight line in a plot of $\ln y$ versus t (see the example). The value of $\ln R_e/r_w$ is dependent on H, D, L, and r_w and can be evaluated from the analog results presented in the next section. The transmissibility T of the aquifer is calculated by multiplying (5) by the thickness D of the aquifer or

$$T = \frac{Dr_c^2 \ln (R_e/r_w)}{2L}\,\frac{1}{t}\ln\frac{y_0}{y_t} \tag{6}$$

This equation is based on the assumption that the aquifer is uniform with depth.

Equations (5) and (6) are dimensionally correct. Thus K and T are expressed in the same units as the length and time parameters in the equations.

EVALUATION OF R_e

Values of R_e, expressed as $\ln R_e/r_w$, were determined with an electrical resistance network analog for different values of r_w, L, H, and D (Figure 1), using the same assumptions as those for (1). An axisymmetric sector of 1 rad was simulated by a network of electrical resistors. The vertical distance between the nodes was constant, but the radial distance between nodes increased with increasing distance from the center line (Figure 2). This yielded a network with the highest node density near the well, where the head loss was greatest, and a decreasing node density toward the outer reaches of the system. For a more detailed discussion of graded networks for representing axisymmetric flow systems, see *Liebmann* [1950] and *Bouwer* [1960].

The radial extent of the medium represented on the analog was more than 60,000 times the largest r_w value used in the analyses. Thus the radial extent of the analog system was essentially infinite, as evidenced by the fact that a reduction in radial extent by several nodes did not have a measurable effect on the observed value of R_e.

The value of R_e for an infinitely deep aquifer ($D = \infty$) was determined by simulating an impermeable and then an infinitely permeable layer at a certain value of D. If this value of D is taken to be sufficiently large, the flow in the system when the layer at D is taken as being impermeable is only slightly

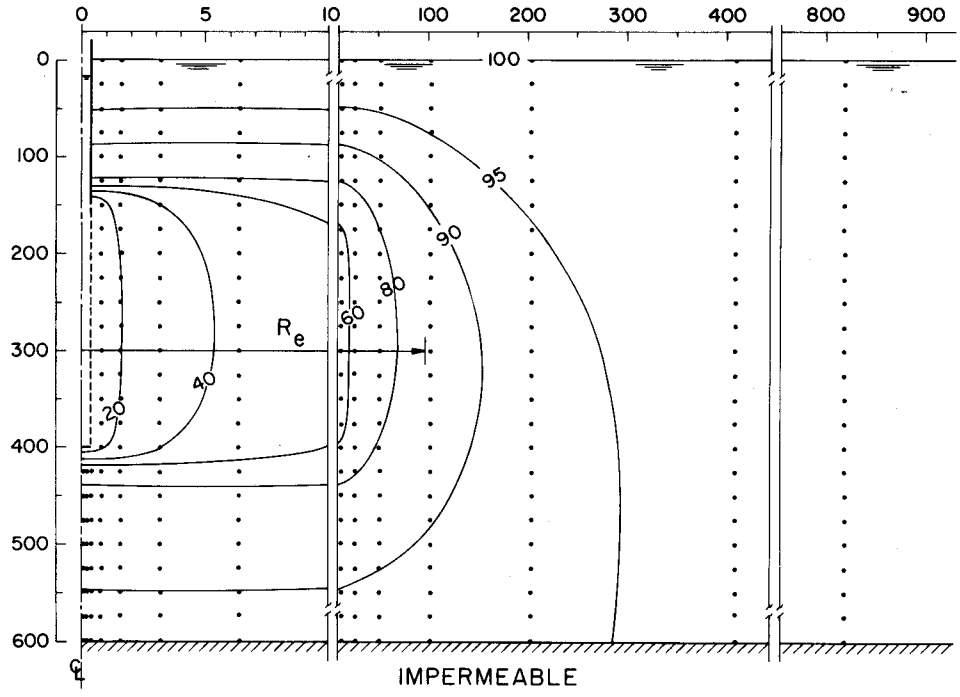

Fig. 2. Node arrangement (dots) for resistance network analog and potential distribution (indicated as percentages on equipotentials) for system with $L/r_w = 625$, $H/r_w = 1000$, and $D/r_w = 1500$. The numbers on the left and at the top of the figure are arbitrary length units (note breaks in horizontal scale).

less than the flow when the layer is taken as being infinitely permeable. The average of the two flows can then be taken as a good estimate of the flow that would occur if the aquifer were represented on the analog as being uniform to infinite depth [*Bouwer*, 1967]. This average flow was used to calculate R_e for $D = \infty$.

The analog analyses were performed by simulating a system with certain values of r_w, H, and D. The electrical current entering the 'well' was then measured for different values of L, ranging from near H to near 0. This was repeated for other values of r_w, H, and D. The condition where $L = H$ could not be simulated on the analog because it would mean a short between the water table as the source and the well as the sink. The electrical current flow in the analog was converted to volume per day, and $\ln R_e/r_w$ was evaluated with (1) for each combination of r_w, H, L, and D used in the analog.

For a given geometry described by r_w, H, and D, the current flow Q_l into the simulated well varied essentially linearly with L and could be described by the equation

$$Q_l = mL + n \tag{7}$$

Because of the linearity between Q_l and L the results of the analyses could be extrapolated to the condition $L = H$. The values of m in (7) appeared to vary inversely with $\ln H/r_w$. The values of n varied approximately linearly with $\ln [(D - H)/r_w]$, the slope A and intercept B in these relations being a function of L/r_w. This enabled the derivation of the following empirical equation relating $\ln R_e/r_w$ to the geometry of the system:

$$\ln \frac{R_e}{r_w} = \left[\frac{1.1}{\ln (H/r_w)} + \frac{A + B \ln [(D - H)/r_w]}{L/r_w}\right]^{-1} \tag{8}$$

In this equation, A and B are dimensionless coefficients that are functions of L/r_w, as shown in Figure 3. If $D \gg H$, an increase in D has no measurable effect on $\ln R_e/r_w$. The analog results indicated that the effective upper limit of $\ln [(D - H)/r_w]$ is 6. Thus if D is considered infinity or $(D - H)/r_w$ is so large that $\ln [(D - H)/r_w]$ is greater than 6, a value of 6 should still be used for the term $\ln [(D - H)/r_w]$ in (8).

If $D = H$, the term $\ln [(D - H)/r_w]$ in (8) cannot be used. The analog results indicated that for this condition, which is the case of a fully penetrating well, (8) should be modified to

$$\ln R_e/r_w = \left(\frac{1.1}{\ln (H/r_w)} + \frac{C}{L/r_w}\right)^{-1} \tag{9}$$

where C is a dimensionless parameter that is a function of L/r_w as shown in Figure 3.

Equations (8) and (9) yield values of $\ln R_e/r_w$ that are within 10% of the actual value as evaluated by analog if $L > 0.4H$ and within 25% if $L \ll H$ (for example, $L = 0.1H$).

The analog analyses were performed for wells that were closed at the bottom. Occasionally, however, wells with open bottoms were also simulated. The flow through the bottom appeared to be negligible for all values of r_w and L used in the analyses. If L is not much greater than r_w (for example, $L/r_w \ll 4$), the system geometry approaches that of a piezometer cavity [*Bouwer and Jackson*, 1974], in which case the bottom flow can be significant. Equations (8) and (9) can also be used to evaluate $\ln R_e/r_w$ if a portion of the perforated or otherwise open part of the well is isolated with packers for the slug test.

Equipotentials for the flow system around a partially penetrating, partially perforated well in an unconfined aquifer after lowering the water level in the well are shown in Figure 2. The numbers along the symmetry axis and the water table represent arbitrary length units. The numbers on the equipotentials indicate the potential as a percentage of the total head difference between the water table (100%) and the open portion of the well (0%) shown as a dashed line.

The value of R_e for the case in Figure 2 is 96.7 length units. As shown in the figure, this corresponds approximately to the

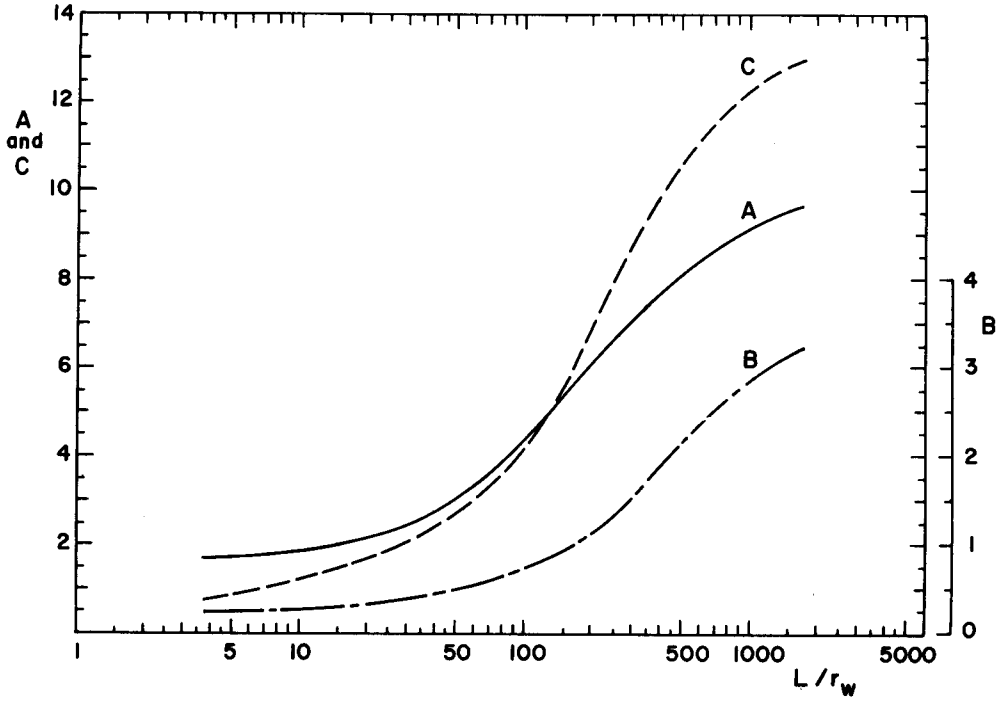

Fig. 3. Curves relating coefficients A, B, and C to L/r_w.

85% equipotential when R_e is laterally extended from the center of the open portion of the well. Thus most of the head loss in the flow system occurs in a cylinder with radius R_e, which is indicative of the horizontal extent of the portion of the aquifer sampled for K or T. The vertical extent is somewhat greater than L, as indicated by, for example, the 80% equipotential in Figure 2.

To estimate the rate of rise of the water level in a well after it is suddenly lowered, (5) can be written as

$$t = \frac{r_c^2}{2KL} \ln \frac{R_e}{r_w} \ln \frac{y_0}{y_t} \tag{10}$$

By taking $y_t = 0.9y_0$, (10) reduces to

$$t_{90\%} = 0.0527 \frac{r_c^2}{KL} \ln \frac{R_e}{r_w} \tag{11}$$

where $t_{90\%}$ is the time that it takes for the water level to rise 90% of the distance to the equilibrium level. By assuming a permeable aquifer with $K = 30$ m/day, a well with $r_c = 0.2$ m and $L = 10$ m, and $\ln (R_e/r_w) = 3$, (11) yields $t_{90\%} = 1.82$ s. Thus if y_0 is taken as 30 cm, it takes 1.8 s for the water level to rise 27 cm, another 1.8 s for the next 2.7 cm (90% of the remaining 3 cm), and another 1.8 s for the next 0.27 cm, or a total of 5.4 s for a rise of 29.97 cm. Measurement of this fast rise requires a sensitive and accurate transducer and a fast-response recorder. The rate of rise can be reduced by allowing groundwater to enter through only a portion of the open section of the well, as can be accomplished with packers.

For a moderately permeable aquifer with, for example, $K = 1$ m/day, a well with $r_c = 0.1$ m and $L = 20$ m, and $\ln (R_e/r_w) = 5$, (11) yields $t = 11.4$ s. In this case, it would take the water level 22.8 s to rise from 30 cm to 0.3 cm below static level.

EXAMPLE

A slug test was performed on a cased well in the alluvial deposits of the Salt River bed west of Phoenix, Arizona. The well, known as the east well, is located about 20 m east of six rapid infiltration basins for groundwater recharge with sewage effluent [*Bouwer*, 1970]. The static water table was at a depth of 3m, $D = 80$ m, $H = 5.5$ m, $L = 4.56$ m, $r_c = 0.076$ m, and r_w was taken as 0.12 m to allow for development of the aquifer around the perforated portion of the casing. A Statham PM131TC pressure transducer was suspended about 1 m below the static water level in the well (when trade names and company names are included, they are for the convenience of the reader and do not imply preferential endorsement of a particular product or company over others by the U.S. Department of Agriculture). A solid cylinder with a volume equivalent to a 0.32-m change in water level in the well was also placed below the water level. When the water level had returned to equilibrium, the cylinder was quickly removed. The transducer output, recorded on a Sargent millivolt recorder, yielded the y-t relationship shown in Figure 4 with y plotted on a logarithmic scale. The straight-line portion is the valid part of the readings. The actual y_0 value of 0.29 m indicated by the straight line is close to the theoretical value of 0.32 m calculated from the displacement of the submerged cylinder.

Extending the straight line in Figure 4 shows that for the arbitrarily selected t value of 20 s, $y = 0.0025$ m. Thus $(1/t) \ln y_0/y_t = 0.238$ s^{-1}. The value of $L/r_w = 38$, for which Figure 3 yields $A = 2.6$ and $B = 0.42$. Substituting these values into (8) and using the maximum value of 6 for $\ln [(D - H)/r_w]$ (since $\ln [(D - H)/r_w]$ for the well exceeds 6) yield $\ln (R_e/r_w) = 2.37$. Equation (5) then gives $K = 0.00036$ m/s $= 31$ m/day. This value agrees with K values of 10 and 53 m/day obtained previously with the tube method on two nearby observation wells [*Bouwer*, 1970]. These K values were essentially point measurements on the aquifer immediately around the well bottoms, which were at depths of 9.1 and 6.1 m, respectively.

COMPARISONS

Piezometer method. The geometry to which (8) and (9) and the coefficients in Figure 3 apply overlaps the geometry of the

piezometer method at the lower values of L/r_w. With the piezometer method a cavity is augered out in the soil below a piezometer tube. The water level in the tube is abruptly lowered, and K of the soil around the cavity is calculated from the rate of rise of the water level in the tube [*Bouwer and Jackson*, 1974]. The equation for K is

$$K = \frac{\pi r_w^2}{A_Y} \frac{1}{t} \ln \frac{y_0}{y_t} \tag{12}$$

where A_Y is a geometry factor with dimension of length. Values of A_Y were evaluated with an electrolytic tank analog by *Youngs* [1968], whose results were expressed in tabular form as A_Y/r_w for different values of L/r_w (ranging between 0 and 8), $(H - L)/r_w$, and $(D - H)/r_w$.

Taking a hypothetical case where $L/r_w = 8$, $H/r_w = 12$, and $D/r_w = 16$, K calculated with (5) is 18% below K calculated with (12). This is more than the 10% error normally expected with (8) and (9) for the L/H value of 0.67 in this case. The larger discrepancy may be due to the difference in methodology, or to the fact that the L/r_w value is close to the lower limit of the range covered on the resistance network analog.

An approximate equation for calculating K with the piezometer method was presented by *Hvorslev* [1951]. The equation, which is based on the assumptions of an ellipsoidal cavity or well screen and infinite vertical extent (upward and downward) of the flow system, contains a term $[1 + (L/2r_w)^2]^{1/2}$. For most well-slug-test geometries, $L/2r_w$ will be sufficiently large to permit replacement of this term by $L/2r_w$. In that case, however, Hvorslev's equation for Q yields $R_e = L$, which is not true. In reality, R_e is considerably less than L. For example, if $L = 40$ m, $r_w = 0.4$ m, $H = 80$ m, and $D = \infty$, (8) shows that $R_e = 11.9$ m, which is much less than the value of 40 m indicated by Hvorslev's equation. However, since the calculation of K is based on $\ln (R_e/r_w)$ as shown by (5), the error in K is less than the error in R_e (i.e., 36 and 236%, respectively, in this case).

If, for the above example, the top of the well screen or cavity had been taken at the same level as the water table ($H = 40$ m), R_e would have been 8.6 m and Hvorslev's equation would have yielded a K value that is 50% higher than K given by (5). The larger error is probably due to Hvorslev's assumption of infinite vertical (upward) extent of the flow system, which is not met when the cavity is immediately below the water table. Using Hvorslev's equation for cavities immediately below a confining layer would increase the error to 73%, but this, of course, is due to the fact that a water table is not a solid boundary. Hvorslev's equation for the confining layer case can be shown to yield $R_e = 2L$.

Auger hole method. The analog analyses for (8) and (9) and Figure 3 were performed for $L < H$, because short circuiting between the water table and the well prevented simulation of the case where $L = H$. If the analog results are extrapolated to $L = H$, however, the geometry of the system in Figure 1 becomes similar to that of the auger hole technique, for which a number of equations and graphs have been developed to calculate K from the rise of the water level in the well [*Bouwer and Jackson*, 1974]. *Boast and Kirkham* [1971], for example, developed the equation

$$K = C_{BK} \frac{\Delta y}{\Delta t} \tag{13}$$

where C_{BK} was determined mathematically and expressed in tabular form for various values of L/r_w, $(D - H)/r_w$, and y_0/H. Since the rate of rise of the water level in the hole after

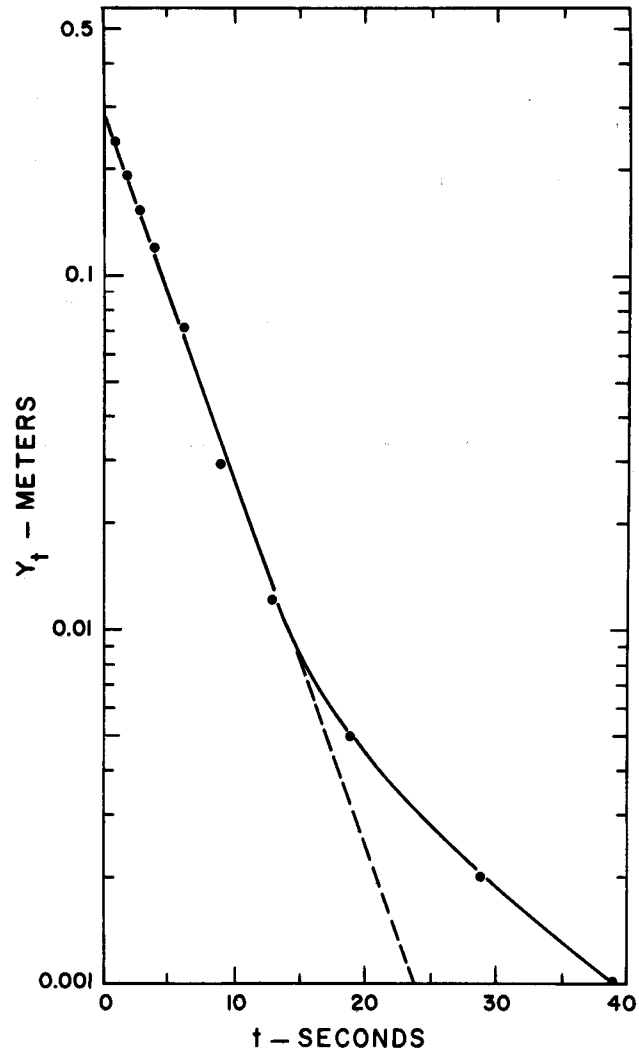

Fig. 4. Plot of y versus t for slug test on east well.

the removal of a slug of water decreases with decreasing y, $\Delta y/\Delta t$ is not a constant and the value of K obtained with this procedure depends on the magnitude of Δy used in the field measurements. The general rule is that Δy should be relatively small.

Taking a hypothetical case where $y_0 = 2.5$ m, $y_t = 2.4$ m, $\Delta t = 10$ s, $L = H = 5$ m, $D = 6$ m, and $r_w = 0.1$ m, (5) yields a K value that is 36% lower than K calculated with (13). However, if y_t is taken as 0.5 m, which should give $\Delta t = 394$ s according to the theory that $(1/t) \ln y_0/y_t$ is constant, the K value yielded by (5) is 26% higher than K obtained with (13). If y_t is taken as 0.9 m, (5) and (13) give identical results.

Slug test on wells in confined aquifers. The confined aquifer for which the slug test by *Cooper et al.* [1967] was developed is an aquifer with an internal water source, for example, recharge through aquitards or compression of confining layers or other material. This situation is similar to that of the unconfined aquifer presented in this paper because the water table is considered horizontal, like the upper boundary of a confined aquifer, and the water table is a plane source. Thus K or T calculated with (5) or (6) should be of the same order as K calculated with the procedure of *Cooper et al.* [1967], which involves plotting the rise of the water level in the well and finding the best fit on a family of type curves. *Cooper et al.* [1967] presented an example of the calculation of T for a well

with $r_c = r_w = 0.076$ m and $L = 98$ m. The resulting value of T was 45.8 m²/day. Values of D and H for this well were not given. However, since the well was 122 m deep and completely penetrating (at least theoretically), D and H must have been between 98 and 122 m. Assuming that both D and H were 100 m, (6) yields $T = 62.8$ m²/day, which is compatible with T obtained by Cooper et al.

CONCLUSIONS

The hydraulic conductivity of an aquifer near a well can be calculated from the rise of the water level in the well after a slug of water is suddenly removed. The calculation is based on the Thiem equation, using an effective radius R_e for the distance over which the head difference between the equilibrium water table in the aquifer and the water level in the well is dissipated. Values of R_e were evaluated by electrical resistance network analog. An empirical equation was then developed to relate R_e to the geometry of the system. This equation is accurate to within 10–25%, depending on how much of the well below the water table is perforated or otherwise open. The technique is applicable to partially or completely penetrating wells in unconfined aquifers. It can also be used to estimate the hydraulic conductivity of confined aquifers that receive water from the upper confining layer through recharge or compression.

The vertical distance between the rising water level in the well and the equilibrium water table in the aquifer must yield a straight line when it is plotted on a logarithmic scale against time. This can be used to check the validity of field measurements and to obtain the best-fitting line for calculating the hydraulic conductivity. Permeable aquifers produce rapidly rising water levels that can be measured with fast-response pressure transducers and strip chart recorders or x-y plotters. The portion of the aquifer sampled for hydraulic conductivity with the slug test is approximately a cylinder with radius R_e and a height somewhat larger than the perforated or otherwise open section of the well.

Hydraulic conductivity values obtained with the proposed slug test are compatible with those yielded by the auger hole and piezometer techniques where the geometries of the systems overlap, and by a slug test for completely penetrating wells in confined aquifers.

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