General Characteristics of Solutions to the Open-Channel Flow, Feedforward Control Problem

E. Bautista, A.M.ASCE1; T. S. Strelkoff, M.ASCE2; and A. J. Clemmens, M.ASCE3

Abstract: A dimensionless formulation of the open-channel flow equations was used to study the feedforward control problem for single-pool canals. Feedforward inflow schedules were computed for specified downstream demands using a gate-stroking model. The analysis was conducted for various design and operational conditions. Differences in the shape of the computed inflow hydrographs are largely related to the volume change resulting from the transient, the time needed to supply this volume, and the time needed by the inflow perturbation to travel down the canal. The gate-stroking method will fail to produce a solution or the solution will demand extreme and unrealistic inflow variations if the time needed to supply the canal volume change is much greater than the travel time of the upstream flow change. As an alternative, a simple feedforward-control flow schedule can be developed based on this volume change and a reasonable delay estimate. This volume compensating schedule can deliver the requested flow change and keep water levels reasonably close to the target under the range of conditions tested.

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Introduction

Development of practical feedforward control strategies that can be applied to a wide range of canal systems and the design of canals that are amenable to feedforward control actions require a thorough understanding of the control characteristics of canals as a function of their physical characteristics and the range of demands imposed on them. Wylie (1969) developed a solution to feedforward control problems, which he named gate-stroking, by solving the governing equations of open-channel flow inversely in space. Chevereau (1991) studied the effect of channel length on the shape of input hydrographs computed with a finite-difference gate-stroking model. Deltour (1992) examined the pool volume changes as a function of changes in flow rate and the target downstream depth (water level setpoint). He developed a series of diagrams for a specific pool that illustrates how pool volume must be adjusted to reach a new steady-state flow under either upstream or downstream control. Using a dimensionless formulation of the de Saint Venant equations, Strelkoff et al. (1998) examined the response of canals to upstream perturbations as a function of canal geometry and downstream boundary conditions. With the nondenimensional formulation, a large range of dimensioned canals can be studied with a single simulation. The study concluded that the shape of the backwater curve is relatively constant for a given dimensionless downstream target depth and channel length (i.e., insensitive to variations in dimensionless bottom width, side slope, and Froude number). Further, they also determined that the downstream boundary condition (e.g., weir, underflow gate, etc.) and backwater depth had a significant impact on how fast an upstream disturbance travels downstream. Bautista et al. (1996) presented a limited analysis of the general features of gate-stroking solutions for single-pool canal systems. That study suggested that the Froude number under the initial flow conditions was an important factor influencing the shape of the computed hydrograph.

This paper extends Bautista’s et al. (1996) analysis of the feedforward control characteristics of canals. The study analyzes the interaction among various design variables on the gate-stroking solutions. Given the limitations of the gate-stroking method, the paper also examines the behavior of downstream water levels when subjected to a simpler feedforward control strategy. The objective is to identify general conditions under which simple anticipatory canal control strategies will yield reasonable control and conditions under which more sophisticated approaches may be required.

Approach

The first part of this paper analyzes the general behavior of inverse (gate-stroking) solutions as a function of canal physical characteristics. The second part compares the effectiveness of the water level control produced by the gate-stroking solution and by a simple feedforward control action based on a simple delay.

Inverse calculations were carried out with an implicit nonlinear finite-difference gate-stroking model (Bautista et al. 1997). Simulations of the response to these hydrographs were computed with the unsteady flow model CanalCAD (Holly and Parrish 1995). Both the finite-difference gate-stroking model and CanalCAD solve the governing equations based on the four-point Pre-
issman finite-difference scheme. A time weighting factor of 0.7 was used in both sets of calculations.

To generalize results, inverse calculations were carried out with the governing equations expressed in dimensionless form. Equations were nondimensionalized using the system of variables proposed by Strelkoff and Clemmens (1998)

\[ A^* = \frac{A}{Y_R}; \quad Q^* = \frac{Q}{Q_R}; \quad X^* = \frac{x}{X_R}; \quad t^* = \frac{t}{T_R} \]  

(1)

where \( A \) = flow area; \( Q \) = discharge; \( x \) = distance along the channel; \( t \) = time; \( Y_R \) = reference length for all transverse canal dimensions; \( Q_R \) = reference discharge; \( X_R \) = reference length for longitudinal dimensions; and \( T_R \) = reference time. The asterisk denotes the dimensionless counterpart of a dimensioned variable. The canal’s design flow \( Q_n \) (or some other convenient flow value) is used to define \( Y_R \) and \( Q_R \). Reference depth \( Y_R \) is set equal to normal depth \( (y_n) \) for \( Q_n \). Thus, \( y_n^* \), the dimensionless normal depth at the design flow, is equal to unity. The dimensionless area at normal depth or aspect ratio, \( A_n^* \), serves to define \( Q_R \) in terms of the design flow

\[ A_n^* = \frac{A_n}{Y_R} \]  

(2)

\[ Q_R = \frac{Q_n}{A_n^*} \]  

(3)

If the channel is trapezoidal, \( A_n^* = b^* + z \), with \( b^* \) the dimensionless bottom width \( (b/Y_R) \) and \( z \) the side slope (horizontal/vertical). It follows from Eq. (3) that \( Q_n^* = A_n^* \) and that the dimensionless normal velocity at design flow \( v_n^* = 1 \). Reference length in the direction of flow \( X_R \) is given by

\[ X_R = \frac{Y_R}{S_{0b}} \]  

(4)

where \( S_{0b} \) = bottom slope at a reference section; and \( X_R \) serves to define the dimensionless canal length \( L^* = L/X_R \). A dimensionless formulation of the de Saint Venant equations that is similar in appearance to the dimensional formulation can be obtained by requiring that

\[ \frac{X_R V^2}{T_R Q_R} = 1 \]  

(5)

where \( T_R \) = reference time. This relationship serves to define the value of \( T_R \). The dimensionless expressions for the governing equations are then

\[ \frac{\partial Q^*}{\partial X^*} + \frac{\partial A^*}{\partial X^*} + Q_0^* = 0 \]  

(6)

\[ \frac{1}{g^*} \left[ \frac{\partial Q^*}{\partial t^*} + \frac{\partial}{\partial X^*} \left( \frac{Q^*}{A^*} \right) + u_0^* Q_0^* \right] + A^* \left[ \frac{\partial h^*}{\partial X^*} + \frac{Q^*}{A^*} \frac{u_0^* Q_0^*}{c_n^*} \right] = 0 \]  

(7)

where \( g^* \) and \( c_n^* \) = dimensionless parameters; \( h^* \) = dimensionless water surface elevation; \( R^* \) = dimensionless hydraulic radius; \( Q_0^* \) = dimensionless lateral outflow per unit length; \( u_0^* \) = dimensionless longitudinal velocity component of the lateral flow; and \( n^* \) = relative Manning \( n, n^* = n n_R \) with \( n_R \) the Manning in a representative canal section. In a uniform canal, \( n^* \) (and \( S_{0b} = S_{0b}/S_{0b} \)) is equal to unity. Expressions for \( g^* \) and \( c_n^* \) are the following:

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
\textbf{F} & \textbf{y} & \textbf{Q} & \textbf{S} & \textbf{T} \\
\hline
0.1 & 123.1 & 5.1 & 3.3E-05 & 45,737 & 36.74 \\
0.15 & 54.7 & 7.6 & 7.4E-05 & 20,327 & 10.89 \\
0.2 & 30.8 & 10.1 & 1.3E-04 & 11,434 & 4.59 \\
0.3 & 13.7 & 15.2 & 3.0E-04 & 5,082 & 1.36 \\
0.4 & 7.7 & 20.2 & 5.2E-04 & 2,859 & 0.57 \\
0.5 & 4.9 & 25.3 & 8.2E-04 & 1,829 & 0.29 \\
0.6 & 3.4 & 30.3 & 1.2E-03 & 1,270 & 0.17 \\
0.7 & 2.5 & 35.4 & 1.6E-03 & 933 & 0.11 \\
0.8 & 1.9 & 40.5 & 2.1E-03 & 715 & 0.07 \\
0.9 & 1.5 & 45.5 & 2.7E-03 & 565 & 0.05 \\
\hline
\end{tabular}
\caption{Hypothetical Dimensioned Channel Characteristics as Function of \textbf{F} \textbf{y} , \textbf{Q} \textbf{y} \textbf{s} , \textbf{S}_{0b} , \textbf{X}_{0b} , and \textbf{T}_{0b}}
\end{table}

\[ g^* = \frac{1}{F^2} \left[ \frac{A_n^*}{B_n^*} \right] \]  

(8)

\[ c_n^* = \frac{1}{R_n^{2/3}} \]  

(9)

where \( F_n = \) Froude number; \( B_n^* = \) dimensionless top width; and \( R_n^* = \) dimensionless hydraulic radius with the subscript \( n \) denoting in all cases normal depth conditions for the design flow. Standard unsteady flow models based on dimensional governing equations similar in form to Eqs. (6) and (7) can be nondimensionalized by replacing the gravitational constant \( g \) with \( g^* \) and the units factor for the Manning equation \( c_n \) with \( c_n^* \). Expressions for other variables in Eqs. (6) and (7) are provided by Strelkoff and Clemmens (1998). A family of hydraulically similar canals is defined by the particular combination of geometric variables, \( b^* \), \( z \), and dimensionless channel length \( (L^*) \) and \( F_n \).

Note that from the definition of wave celerity \( c \) (Henderson 1966)

\[ c = \sqrt{g A/B} \]  

(10)

the celerity at normal depth for the design flow, \( c_n^* \), is the same for all canals with the same aspect ratio. In Eq. (10), \( B \) represents the top width. Its dimensionless counterpart, \( c_n^* \), is simply the inverse of \( F_n \)

\[ c_n^* = \frac{c_n}{v_n} = \frac{1}{F_n} \]  

(11)

### Gate-Stroking Solutions

#### Effect of Froude Number \( F_n \)

The analysis is restricted to single-pool canals of uniform cross section and slope. The effect of variable \( F_n \) on the gate-stroking solutions is analyzed first for canals with dimensionless characteristics \( b^* = 5, z = 1.5, \) and \( L^* = 1.0 \). The impact of \( b^* \), \( z \), and \( L^* \) will be examined later. With this choice of \( b^* \) and \( z \), \( A_n^* \) and \( Q_n^* \) are both equal to 6.5 [Eq. (3) and comment following]. For illustration purposes, hypothetical dimensioned channels (Strelkoff and Clemmens 1998) were computed by assuming \( Y_R = 1.5 \) m and \( n = 0.018 \) (Table 1). The dimensioned channels have identical
cross sections at normal depth (area, top width, wetted perimeter, etc.) but differ in length, slope, design capacity, etc. as a function of $F_n$.

The example seeks to route a single stepwise change in downstream demand $\Delta Q^*_o$ in a channel that is initially under steady state, while keeping the downstream water depth $y^*_o$ at the set-point $y^*_o = 1.25$ (1.25 times the design flow normal depth). Initial inflow $Q^*_o$ is 90% of $Q^*_o$ (5.85); $\Delta Q^*_o$ is 10% of $Q^*_o$ (0.65); and the time at which the demand change takes place, $t^*_D$, is 1.0. In the following paragraphs, the subscript $o$ denotes the value of the variable under the initial flow conditions. To facilitate the discussion, results are presented in terms of relative discharge $Q^*_o$, which is related to $Q^*$ as follows:

$$Q^*_o = \frac{Q^*_o}{Q^*_o} = \frac{Q}{Q^*_o}$$  \hspace{1cm} (12)

Therefore, the initial and final relative inflow $Q^*_o$ are 0.9 and 1.0, respectively.

Numerical experiments were conducted to determine an appropriate combination of spatial and temporal increments, $\Delta x^*$ and $\Delta t^*$, respectively, in the finite difference solutions. Nearly identical results were obtained with $\Delta x^* = 0.025$ and, thus, 0.025 was used as the space increment in all subsequent calculations. $\Delta t^*$ was determined by enforcing a dimensionless Courant condition close to 1, based on the selected $\Delta x^*$ and $F_n$ (and thus $c^*_o$) to minimize numerical damping effects on the solution.

For the proposed flow conditions, the desired outflow hydrograph becomes physically more difficult to produce as $F_n$ decreases, requiring more extreme variations in the computed inflow hydrograph (Fig. 1). Peak upstream $Q^*_o$ at $F_n = 0.15$ is nearly three times the canal capacity, which is an unacceptable solution in practice. For $F_n < 0.15$, no solution could be found as the execution was terminated due to the calculation of negative $A^*$ values. This implies that for $F_n < 0.15$, inflow cannot be varied in any way to produce the desired output. Peak inflow decreases rapidly with increasing $F_n$, and is nearly equal to the desired final demand for $F_n > 0.5$. Results also indicate that the transient needs to be initiated ever earlier, in dimensionless time, with increasing $F_n$. If the duration of the gate-stroking computed transient (which will be denoted by $\tau^*_D$) is defined as the time between the initial upstream flow change and the time at which the demand change occurs, then from Fig. 1, $\tau^*_D$ is about 2.9 times greater for $F_n = 0.9$ than for $F_n = 0.15$ (Table 2).

![Fig. 1. Variation in gate-stroking solutions with $F_n$ ($L^* = 1$, $b^* = 5$, $z = 1.5$, $y^*_o = 1.25$)](image)

It is useful to analyze the role of $F_n$ in determining the solutions of Fig. 1. Initial conditions used to solve Eqs. (6) and (7) are nearly the same for each value of $F_n$, and boundary conditions are exactly the same. Thus, differences in the computed hydrographs can be attributed mostly to the parameter $g^*$ in Eq. (8). $g^*$ increases as $1/F_n^2$, and this magnifies the relative contribution of the local and convective acceleration terms in Eq. (7).

The role of $F_n$ can be analyzed also in terms of volume changes needed to produce the transient, the time needed to supply this volume, and the time needed for the perturbation to travel down the canal. Because $F_n$ has little influence on the shape of the dimensionless, steady-state water surface depth and velocity profiles, $\Delta V^*$, the dimensionless volume change from the initial steady state to the final steady state, varies slightly with $F_n$ ($\Delta V^*$ represents in all cases close to 4% of the initial volume $V^*_o$) (Table 2). If $\Delta Q^*_o$ is used to supply $\Delta V^*$, then the dimensionless time needed to supply that change is $\tau^*_D V^*$:

$$\tau^*_D V^* = \frac{\Delta V^*}{\Delta Q^*_o}$$  \hspace{1cm} (13)

This estimate assumes the canal outflow remains constant up to $t^* = t^*_D$. Because $\Delta Q^*_o$ is a constant (0.65), $F_n$ has little influence also on $\tau^*_D V^*$ (Table 2). $F_n$ has an important effect on the travel time, however. An estimate of the minimum dimensionless time needed by the leading edge of the flow perturbation to travel down the canal is $\tau^*_D$:

$$\tau^*_D = \frac{L^*}{v^*_o + c^*_o}$$  \hspace{1cm} (14)

where $v^*_o$ and $c^*_o$, respectively, are average dimensionless velocity and celerity under the initial flow conditions. Their sum is the average dimensionless dynamic wave velocity. $v^*_o$ is nearly constant with $F_n$, but $c^*_o$ varies essentially with $1/F_n$. (The values in Table 2 were computed based on $v^*_o$, the average, dimensionless, initial depth of flow.) Therefore, $\tau^*_D$ is about 3.8 times greater at $F_n = 0.9$ than at $F_n = 0.15$ (Table 2).

If $\tau^*_D V^* > \tau^*_D$, as occurs at lower values of $F_n$, then discharge can be increased to supply $\Delta V^*$ within the time required by the inflow perturbation to travel. Under those conditions, flow must be accelerated and subsequently decelerated to keep the water level at the downstream end of the canal constant. However, if the ratio $\tau^*_D V^*/\tau^*_D$ is close to 1, as occurs at high values of $F_n$, then $\Delta Q^*_o$ can supply the needed $\Delta V^*$ in about the same time required.
for the perturbation to travel down the canal and flow rate does not need to be increased beyond $\Delta Q^*_R$. In summary, for a given flow geometric configuration ($L^*, b^*, z*$, and $y^*_m$), as $F_n$ increases, the transient goes from being limited by the volume change to being limited by the dynamic characteristics of the canal.

It is helpful to review these relationships in dimensional form. With $\Delta V^*$ nearly constant, differences in actual $\Delta V$ as a function of $F_n$ are essentially explained by differences in the scaling factor $X_R$ (Table 1), which varies with $1/F_n^2$. Thus, volume change is nearly 36 times greater at $F_n=0.15$ than at $F_n=0.9$. At the same time, the flow rate available to supply this volume change decreases with decreasing $F_n$ (Table 1). As a result, it takes almost 216 times longer to supply this volume at $F_n=0.15$ than at $F_n=0.9$ (Table 2).

### Table 3. Hypothetical Dimensioned Channel, and Dimensionless Volume Change and Delay Characteristics of Computed Transient as Function of $b^*$ for Channels with $F_n=0.15$ and $F_n=0.5$ ($L^*=1$, $z^*=1.5$): Dimensioned Channels Computed with $Y_R=1.5$ m, and $n=0.018$

<table>
<thead>
<tr>
<th>Variable</th>
<th>$F_n=0.15$</th>
<th>$F_n=0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b^*$</td>
<td>$b^*$</td>
</tr>
<tr>
<td>$F_R^2$</td>
<td>0.016 0.018 0.020</td>
<td>0.182 0.203 0.221</td>
</tr>
<tr>
<td>$g^*$</td>
<td>61.11 54.70 50.24</td>
<td>5.50 4.92 4.52</td>
</tr>
<tr>
<td>$Q_e$ (m$^3$/s)</td>
<td>4.42 7.59 14.00</td>
<td>14.72 25.29 46.68</td>
</tr>
<tr>
<td>$S_{D_2} (\times 10^{-4})$</td>
<td>0.80 0.89 0.97</td>
<td>8.87 9.91 10.8</td>
</tr>
<tr>
<td>$X_R$ (m)</td>
<td>18,785 16,814 15,444</td>
<td>1,692 1,513 1,390</td>
</tr>
<tr>
<td>$T_R$ (h)</td>
<td>10.63 9.01 7.93</td>
<td>0.29 0.24 0.21</td>
</tr>
<tr>
<td>$\Delta V^*$</td>
<td>0.19 0.29 0.48</td>
<td>0.19 0.3 0.49</td>
</tr>
<tr>
<td>$\tau_{DV}^*$</td>
<td>0.47 0.44 0.41</td>
<td>0.48 0.45 0.43</td>
</tr>
<tr>
<td>$\tau_{D^2}^*$</td>
<td>0.09 0.11 0.12</td>
<td>0.35 0.35 0.35</td>
</tr>
<tr>
<td>$\tau_{DV}^* / \tau_{D^2}^*$</td>
<td>5.21 4.21 3.59</td>
<td>1.39 1.31 1.24</td>
</tr>
</tbody>
</table>

### Effect of Dimensionless Bottom Width $b^*$ and Side Slope $z$

Tables 3 and 4 show the hypothetical dimensioned channel for two values of $F_n(0.15,0.5)$ as a function of, respectively, dimensionless bottom width and canal side slope. The range of $b^*$ and $z$ values considered is typical of real canals. While $b^*$ and $z$ are transverse scale factors, they also affect the longitudinal, time, and flow rate scale of the problem. As the channel widens with other dimensionless design and operational variables held constant ($L^*=1$ and $y^*_m=1.25$), the desired downstream outflow hydrograph becomes easier to produce (Fig. 2). Note that in this figure as in Fig. 3, the left axis represents the scale for $F_n=0.15$ while the right axis is the scale for the $F_n=0.5$ curves. Even though $\Delta V^*$ increases by 2.5 times in the range of $b^*$ studied, $\tau_{DV}^*$ decreases (as a result of increasing design capacity) while $\tau_{D^2}^*$ increases, at $F_n=0.15$, or decreases slightly, at $F_n=0.5$ (Table 3). In contrast, increasing $z$ makes control more difficult (Fig. 3). In this case, $\tau_{DV}^*$ increases (the capacity increase is insufficient to offset the volume change) while $\tau_{D^2}^*$ decreases at either $F_n$ value (Table 4). Differences in computed hydrograph shapes are more modest for varying $z$ than for varying $b^*$ within the typical range of interest.

### Effect of Dimensionless Length $L^*$

Figs. 4 and 5 illustrate the effect of $L^*$ on the gate-stroking solutions for two values of $F_n$, 0.15 and 0.5 (with $b^*=5$ and $z^*=1.5$) with $y^*_m=1.25$, as in the initial example. For $F_n=0.5$ and $L^*>2$, inflow variations had to be initiated earlier than $t_D^*=0$ to complete the transient by $t_D^*=2$, inflow perturbation could, perhaps, attenuate rapidly and then travel at nearly constant speed, i.e. as a kinematic wave (Henderson 1966; Fenton et al. 1999). This theory was tested by computing values of $v^*_K$ and $\tau_{DG}^*$ (defined in an earlier section) varies almost in proportion to $L^*$. This suggests that the upstream inflow perturbation could, perhaps, attenuate rapidly and then travel at nearly constant speed, i.e. as a kinematic wave (Henderson 1966; Fenton et al. 1999). This theory was tested by computing values of $v^*_K$, the dimensionless kinematic wave velocity, and the resulting travel time estimates $\tau_{KG}^*=L^*/v^*_K$, as a function of $L^*$ for the average initial depth conditions (Table 5). $v^*_K$ is given by

$$v_{K}^* = \frac{dx^*}{dt^*} = \frac{1}{B^*} \frac{dQ^*}{dy^*}$$

Given in the same table are estimates of $\tau_{DG}^*$ (derived from Figs. 4 and 5). $v_{K}^*$ increases only slightly with $L^*$ but the resulting travel time $\tau_{KG}^*$ is much greater than $\tau_{DG}^*$, over 4.3 times at $F_n=0.15$ and over 1.8 times at $F_n=0.5$. Consequently, and because a closer relationship exists between $\tau_{DG}^*$ and $\tau_{D^2}^*$, one would have to conclude that the inflow perturbation does not behave like a kinematic wave, especially in the lower $F_n$ range.
The effect of increasing $y_{st}^*$ on the gate-stroking solutions was investigated also, using two values of $F_n = 0.15$ and 0.5 (normal depth at design capacity) to 25% above normal depth reduces $\Delta V^*$ by 10% (Table 6); consequently, peak $Q^*_{rel}$ decreases by over 30% (Fig. 6). While $\Delta V^*$ continues to decrease as $y_{st}^*$ increases, substantial increases in $y_{st}^*$ are needed to force peak $Q^*_{rel}$ close to canal capacity. Increasing $y_{st}^*$ also reduces $t_{GS}^*$ (the transient’s duration) (Fig. 6), even though $t_{DW}^*$ remains nearly constant (Table 6). Results computed at the higher value of $F_n$ show a similar variation in $\Delta V^*$ with $y_{st}^*$ (Table 6). However, because small inflow changes are already required at $y_{st}^* = 1$, increasing $y_{st}^*$ only has a slight impact on peak $Q^*_{rel}$ (Fig. 7). Similarly, the effect on the transient’s duration is minimal. In Fig. 7, for the gate-stroking hydrographs computed at $y_{st}^* = 1.75$ and 2.0, inflow rate during the transient never exceeds final $Q^*_{rel}$ and this value is reached only after $t^* = 1.5$ (that is, after the final downstream outflow conditions have been achieved). In these cases, the ratio $\tau_{GV}^*/\tau_{DW}^*$ is less than 1 (Table 6), so the only limitation to producing the desired demand change is the velocity at which the perturbation can travel down the canal.

**Forward Solutions**

Simulations were conducted with CanalCAD to test the gate-stroking solutions presented in Fig. 1. Since CanalCAD does not allow the user to modify the values of $g$ and $c_u$, calculations were carried out in dimensional form, using the hypothetical dimensional channel data (Table 1). Results were subsequently nondimensionalized. Simulations used the gate-stroking solution as an upstream boundary condition. The downstream boundary condition consisted of a vertical sluice gate at a fixed position. The gate opening was determined assuming free flow based on the gate width $b^*$ (5), $Q^*_o = 5.85$, and the given $y_{st}^*$ value (1.25). A gravity offtake just upstream from the check structure was used to simulate the demand change. The offtake was initially closed and was opened at $t^* = 1$ to deliver $\Delta Q^*_o$. The constant offtake opening was calculated internally by the simulator based on the desired depth upstream from the offtake, $y_{st}^*$, and a constant depth downstream from the offtake equal to half normal depth. The operational scenario represented by this gate and turnout combination is realistic although results cannot be fully generalized because of the specific gate and turnout hydraulic relationships used in CanalCAD.
The time variation in downstream water depth deviations, \( y^*(t^*) - y^*_{\text{ref}} \), is illustrated in Fig. 8. The desired initial steady-state flow conditions could not be matched due to round-off errors. These errors were slight in most cases but increased abruptly at \( F_n = 0.9 \). While the gate-stroking solutions were unable to keep the downstream depth perfectly stable throughout the transient, control was excellent, with peak deviations not exceeding 0.03 (less than 3% of normal depth), and results improved as \( F_n \) decreased. Note that the inability of the gate-stroking solution to produce the desired transient is due to theoretical limitations of the gate-stroking concept (Fenton et al. 1999; Bautista et al. 1997) and round-off errors of both the inverse and forward solutions.

The results presented in previous sections showed how the dimensionless design and operational variables interact to determine the shape of the gate-stroking solution. For a given demand change, some conditions require substantial inflow acceleration and deceleration; for other conditions, inflow changes essentially cause it explicitly accounts for the pool volume storage change needed for the new steady-state condition. Volume compensation is one of the principles behind the dynamic regulation canal control method (Deltour 1992). Note that this compensating volume is supplied also by the gate-stroking solutions. Feedforward schedules were developed using Eq. (16), the \( \Delta V^n \) and \( \tau^n_{SV} \) values of Table 2, and the specified \( \Delta Q^n_d \). Simulations were conducted with the same upstream boundary condition and off-take configuration as before. Depth deviations obtained with this feedforward control strategy (Fig. 9) were only slightly larger than those obtained with the gate-stroking solutions (Fig. 8). More important, reasonable results were obtained even for \( F_n = 0.15 \), conditions under which the gate-stroking solution required significant acceleration and deceleration of flow.

In Fig. 9, all deviations are positive. This implies that most of the volume change needed for the new steady-state, has been stored in the pool by the time the off-take is opened. Further reductions in the downstream water depth deviations seem possible, therefore, by using the same compensating volume \( \Delta V^n \) but with a smaller delay \( \tau^* \), i.e., by timing the upstream inflow change so that the downstream flow rate change occurs before the bulk of the volume change has been added to the pool. A reasonably small choice for \( \tau^* \) is \( \tau^*_{\text{DW}} \). The resulting feedforward schedule consists then of two upstream flow changes:

\[
\Delta Q^n_1(t^n_1) = \frac{\Delta V^n}{\tau^n_{\text{DW}}} ; \quad t^n_1 = t^n_d - \tau^n_{\text{DW}}
\]

(17)

The second change is needed to match the inflow with the sum of the initial inflow and the demand flow change. Volume compensating schedules were developed with Eq. (17) for the same

### Table 5. Dimensionless Volume Change and Delay Characteristics of Computed Transient as Function of \( F_n \) and \( L^* \)

<table>
<thead>
<tr>
<th>( F_n = 0.15 )</th>
<th>( F_n = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L^* )</td>
<td>( L^* )</td>
</tr>
<tr>
<td>( \Delta V^n )</td>
<td>0.25</td>
</tr>
<tr>
<td>( \tau^n_{SV} )</td>
<td>0.02</td>
</tr>
<tr>
<td>( y^*_0 )</td>
<td>0.04</td>
</tr>
<tr>
<td>( \tau^n_{DW} )</td>
<td>1.18</td>
</tr>
<tr>
<td>( \tau^n_{\text{FC}} )</td>
<td>0.03</td>
</tr>
<tr>
<td>( \tau^n_{\text{GS}} )</td>
<td>0.12</td>
</tr>
<tr>
<td>( \tau^n_{\text{WC}} )</td>
<td>1.01</td>
</tr>
<tr>
<td>( \tau^n_{\text{KWC}} )</td>
<td>0.25</td>
</tr>
<tr>
<td>( \Delta V^n )</td>
<td>0.06</td>
</tr>
</tbody>
</table>

### Table 6. Dimensionless Volume Change and Delay Characteristics of Computed Transient as Function of \( F_n \) and \( y^*_s \)

<table>
<thead>
<tr>
<th>( F_n = 0.15 )</th>
<th>( F_n = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y^*_s )</td>
<td>( y^*_s )</td>
</tr>
<tr>
<td>( \Delta V^n )</td>
<td>0.33</td>
</tr>
<tr>
<td>( \tau^n_{SV} )</td>
<td>0.51</td>
</tr>
<tr>
<td>( \tau^n_{\text{DW}} )</td>
<td>0.13</td>
</tr>
<tr>
<td>( \tau^n_{SV}/\tau^n_{\text{DW}} )</td>
<td>3.83</td>
</tr>
</tbody>
</table>

Fig. 7. Variation in gate-stroking solutions with \( y^*_s \) at \( F_n = 0.5 \) (\( L^* = 1, b^* = 5, z^* = 1.5 \))

Fig. 6. Variation in gate-stroking solutions with \( y^*_s \) at \( F_n = 0.15 \) (\( L^* = 1, b^* = 5, z^* = 1.5 \))
flow conditions as before, using the $\tau_{DW}^*$ values of Table 2. Water level deviations were mostly negative at lower $F_n$ values, indicating that $\tau_{DW}^*$ underestimates the delay under those conditions (Fig. 10). Slightly better results were obtained at higher values of $F_n$, even though $\tau_{DW}^*$ and $\tau_{DW}^*$ are very similar under those conditions (Table 2). This suggests the volume compensating approach is fairly sensitive at high Froude numbers. Despite these limitations, these results along with those obtained with Eq. (16) show that a feedforward strategy based on volume compensation will perform adequately if the delay is within $\tau_{DW}^*$, for cases in which $\tau_{DW}^* < \tau_{DV}^*$. Tests not presented here suggest that in the alternative case, $\tau_{DW}^* < \tau_{DV}^*$ (e.g., the solution computed for $F_n = 0.5$ and $y_{st} = 1.75$ in Table 6), better water level control would be obtained with $\tau_{DW}^*$ as the delay. It is worth noting that in a previous study, two of the authors (Bautista and Clemmens 1999) proposed computing a delay for feedforward control based on a combination of $\tau_{DW}^*$, computed for the pool section under backwater influence, and $\tau_{KW}$ (see text preceding Eq. (16)) applied to the section not affected by backwater. Results presented herein suggest that approach, besides being more complex, could severely overestimate the wave travel time, particularly at very low Froude numbers and when flow is close to normal.

Additional simulation tests were conducted with one of the examples shown in Fig. 5, a canal with design $F_n = 0.5$ and $L^* = 4.0$. As shown in Fig. 5, the gate-stroking solution for this case requires large acceleration and deceleration of flow. Simulations used first the gate-stroking solution, and then the volume compensating schedule with delay $\tau_{DV}^*$. Peak downstream water depth deviation computed with the gate-stroking solution for $L^* = 4$ (Fig. 11) is only slightly larger than that obtained for the same $F_n$ value but with $L^* = 1$ (Fig. 8). Thus, the gate-stroking method continued to produce reasonable results under these more extreme flow change conditions. In contrast, peak depth deviation computed with the volume compensating schedules at $L^* = 4$ (Fig. 1) is about 1.6 times greater than the corresponding results obtained at $L^* = 1$ (Fig. 9). Clearly, the effectiveness of the volume compensating decreases with increasing channel length although results may still be adequate for field applications. As in Fig. 9, deviations obtained with the volume compensating schedule are all positive, meaning that control can be further improved through the choice of a smaller delay.

**Discussion**

Gate-stroking is a restrictive method, because it requires downstream depth to vary in a prespecified manner. It is also a poorly-proposed problem (Cunge et al. 1980). As a result, solutions are very sensitive to the physical characteristics of canals. Despite these limitations, the above presented analysis provides us with insights about the feedforward control problem.

Clearly, accounting for the change in pool storage needed for the new steady state is critical, independently of canal characteristics. Both the gate-stroking method and the proposed simple scheme account for this change and are nearly as effective. For a given $F_n$, gate-stroking solutions become more extreme with increasing $L^*$ and decreasing setpoint, i.e., as supply time becomes greater relative to travel time. These are also conditions under which control with the simple volume-compensating feedforward scheme degrades. While the resulting downstream water level deviations are more extreme, they are still small relative to the setpoint and would not endanger the canal. Also, while unsteady conditions persist for longer times than with gate-stroking, the deviations would only have a slight effect on the accuracy of deliveries. Thus, the degradation in water level control would not offset the benefit less extreme inflow fluctuations which are potentially required with the gate-stroking solution. The analysis also shows that time at which gate-stroking inflow changes need to be initiated are generally bound by simple travel time estimates, $\tau_{DW}$ and $\tau_{DV}$. Thus, simple scheduling approach can be developed in accordance with these bounds.
The analysis presented herein applies to single pool canals of uniform cross section subject to a single demand change. Real irrigation delivery systems consist of networks of canals, with pools of irregular slope and cross section, and multiple flow changes. A canal scheduling algorithm based on volume compensation for complex delivery systems has been proposed (Bautista and Clemmens 1999) and is currently being further developed.

Conclusions

Differences in the shape of inflow hydrographs computed with the gate-stroking method can be explained in terms of volume changes resulting from the transient, the time needed to supply this volume, and the time needed by the inflow perturbation to travel down the canal. Severe inflow fluctuations are needed when the ratio between the supply and travel time is relatively large, that is when the supply rate limits the desired flow change. This relationship is Froude number dependent and, therefore, it is difficult to determine whether significant flow increases will be needed to produce a desired downstream flow change from that ratio alone.

A feedforward control strategy based on volume compensation performed nearly as well as the gate-stroking method under the range of conditions studied. The method is simple as it requires at most two discrete flow rate changes and not continuous flow variations as with gate-stroking, and the magnitude of required flow changes can be bound. Thus, the method represents a practical feedforward control strategy. Reasonable bounds for the delay in the volume compensating method are the time needed to supply the volume change and the travel time computed from dynamic wave theory. Within this range, errors in the delay estimation will result in moderate deviations over relatively long times at lower values of the flow’s Froude number, and larger but shorter-lived deviations at higher Froude values.

Notation

The following symbols are used in this paper:

- \( A \) = flow area;
- \( A_n \) = flow area at normal depth for the design flow (aspect ratio);
- \( B \) = top width;
- \( b \) = bottom width;
- \( c \) = celerity;
- \( c_o^* \) = average dimensionless celerity under the initial flow conditions;
- \( c_u \) = units parameter in the Manning resistance equation;
- \( c_u^* \) = parameter in the dimensionless momentum equation;
- \( F_n \) = Froude number at normal depth for the design flow;
- \( g \) = gravitational constant;
- \( g^* \) = parameter in the dimensionless momentum equation;
- \( h \) = water surface elevation;
- \( L \) = channel length;
- \( n \) = Manning n;
- \( Q \) = flow rate;
- \( Q_n \) = design flow rate;
- \( Q_R \) = reference flow rate;
- \( Q_{nl}^* \) = relative flow rate;
- \( q_0 \) = lateral flow per unit length;
- \( R_n \) = hydraulic radius at normal depth for the design flow;
- \( S_0 \) = channel bottom slope;
- \( S_{ok} \) = reference slope;
- \( T_R \) = reference time;
- \( t \) = time;
- \( \tau_t \) = time for the demand flow rate change;
- \( v_o^* \) = average dimensionless flow velocity under the initial flow conditions;
- \( x_R \) = reference length along the channel;
- \( x \) = length along the channel;
- \( y_R \) = reference transverse length;
- \( y \) = flow depth;
- \( y_{st} \) = water depth setpoint;
- \( y_o^* \) = dimensionless average depth of flow under the initial flow conditions;
- \( z \) = canal side slope;
- \( \Delta Q_d \) = demand flow rate change;
- \( \Delta x^* \) = dimensionless space increment in the finite difference solution;
- \( \Delta t^* \) = dimensionless time increment in the finite difference solution;
- \( \Delta V \) = volume change between the initial and final steady-state flow rate;
- \( \tau \) = delay;
- \( \tau_{DW} \) = dynamic wave delay;
- \( \tau_{GS} \) = duration of the gate-stroking transient; and
- \( \tau_{AV} \) = time needed to supply \( \Delta V \).

Superscript

- * = dimensionless counterpart of a dimensioned variable.

References


