Crops consume water in the process of transpiration, and water evaporates from the soil. These processes are defined collectively as evapotranspiration (Thornthwaite, 1948). Only the transpiration portion of evapotranspiration directly influences crop production (de Wit, 1958). Although soil evaporation can be reduced, it is practically impossible to totally eliminate soil water evaporation, even with expensive plastic or artificial mulches (Klocke et al., 1985). There has been a long history dating back to the late 17th century (Woodward, 1699) of efforts to determine water use by crops and vegetation. The necessary amount of irrigation water for crop production has been of interest to investigators at least since the 19th century in the USA (Mead, 1887).

The purpose of this chapter is to summarize the voluminous information on the relationship between crop production and evapotranspiration. Soil water evaporation, deep percolation, runoff, and soil water recharge can result from irrigation but may not directly increase crop production. Previous chapters have discussed the physical and biological limitations to the evapotranspiration process, and this chapter will focus on the crop production associated with transpiration and evapotranspiration with an extension to the relationship between crop production and applied irrigation water. Past reviews of this subject are Doorenbos and Kassam (1979), Hanks and Rasmussen (1982), Taylor et al. (1983), Stanhill, 1986), and van Keulen and Wolf (1986).

The quantity of irrigation water necessary for crop production has been historically important, particularly in the arid western USA. The water right granted to an irrigator as a result of prior appropriation or adjudication was called the duty of water (Powers, 1922). The term duty of water was widely used throughout the late 19th century and is still in use (Lety & Vaux, 1984). The duty of water was the amount of water required to be diverted to irrigate a crop area sufficiently to produce an economic yield. The term consumptive use (ASCE, 1930) was used beginning in the early 20th century and was defined as the evapotranspiration of the crop and has largely replaced
the term duty of water in legal institutions. Early research defined the terms water requirement and transpiration ratio (Briggs & Shantz, 1913a, 1913b) to mean the ratio of the amount of transpiration (usually expressed in units of mass) to the production of crop dry matter (usually expressed in units of mass and excluding the root mass). Transpiration ratio was also known as the transpiration coefficient (Maximov, 1929). Viets (1962) defined water use efficiency as the ratio of the crop production to evapotranspiration. Water use efficiency has become a widely used agronomic term implying the yield (photosynthesis, biological, or economic) per unit of water (transpiration, evapotranspiration, or applied water). Sinclair et al. (1984) classified the water use efficiency terminology for several production measurements (photosynthesis to yield) for different time scales.

The agronomic or physiological characterization of water use efficiency is defined differently than the engineering definition in which water use efficiency means the ratio between the amount of water stored in the crop root zone to the amount delivered for irrigation (Bos & Nugteren, 1978). The engineering characterization of water use efficiency is normally expressed as a volume percentage. This chapter is associated with the agronomic view of water use efficiency as contrasted with the engineering view. When the agronomic value of water use efficiency is increased, the engineering value of water use efficiency will likely be improved, although maybe not in a direct proportion.

Clearly, the relationship between crop production and the amount of irrigation water applied to the crop is important to agronomists, engineers, economists, and water resource planners. This importance is currently accentuated due to competition among users, declining groundwater reserves, various legal institutions, and degradations in water quality. The relationship between crop production and irrigation applications is not unique and is often not clearly defined.

Crop production models with resource and management inputs (as input-output models) have been widely used, particularly by agricultural economists, and called production functions (Hexem & Heady, 1978; Vaux & Pruitt, 1983). These production functions have permitted analyses of resource problems, usually in terms of one or two inputs. Agricultural production depends on many resource or managerial inputs, in addition to irrigation or rainfall, that may not be properly characterized in such one- or two-dimensional systems. The relationship of crop production to irrigation also depends on the salinity of the soil and irrigation water, the uniformity of the irrigation applications, the spatial variability of the soil physical properties, specific crop variety characteristics, and crop cultural practices (e.g., weed and pest control, fertility, plant population, row spacing, and planting date).

I. ANALYTICAL CONCEPTS

The analytical framework for describing the effects of irrigation on crop yield is complex. This chapter will discuss the effects of many parameters
on crop yield through their effects on several processes. These processes (mainly assimilation, transpiration, evapotranspiration, etc.) are discussed in other chapters in this monograph. The framework for this chapter is based on understanding the dynamic nature of the following relationships:

$$ET = f(Q, \theta, C, W, M),$$

$$T = f(ET, C, W, M),$$

$$A = f(T, C, W, M)$$

$$P = f(A, C, M)$$

and $$Y = f(P, C, M)$$

where \( f \) represents a functional relationship between many specific production vectors, \( ET \) is evapotranspiration, \( Q \) is irrigation, \( \theta \) is various soil vectors (water content, nutrient content, salinity, etc.), \( C \) is various crop vectors (species, diffusion resistances, \( CO_2 \) compensation point, and partitioning), \( W \) is various weather vectors (solar radiation, air temperature, vapor pressure deficit, rainfall, etc.), \( M \) is various miscellaneous vectors (diseases, critical water deficit periods, insects, agronomic culture, etc.), \( T \) is transpiration, \( A \) is assimilation, \( P \) is dry matter production, and \( Y \) is economic yield. (These and other symbols used in this chapter are listed and explained in the Appendix.) The development of the complete functional relationships in this framework would be extremely difficult, if not impossible. Even when simple relationships between the various vectors have been developed, the integration of all the factors related to crop yield remains complex.

If the costs of production are neglected for the moment, the goal of most agronomic systems is to produce the most yield subject to the physical and chemical limitations expressed by the above equations. Improving engineering water use efficiency relates to maximizing \( ET \) from \( Q \). Improving agronomic water use efficiency \( (Y \cdot ET^{-1}) \) relates to maximizing the yield partitioning (harvest index, yield structural components, etc.), minimizing the transpiration ratio \( (T \cdot A^{-1}) \), and maximizing the partitioning of transpiration from evapotranspiration (Viets, 1962; Tanner & Sinclair, 1983; Cooper et al., 1987). This section will discuss this analytical framework within the current level of understanding for the interactions of the specific production vectors.

A. Assimilation-Transpiration Relationships

The mean rate of leaf transpiration is given as

$$T = (W_s - W_a) (\Sigma r)^{-1}$$

where \( T \) is the transpiration rate in kg \( (H_2O) \) m\(^{-2}\) s\(^{-1}\), \( W_a \) is the atmospheric water vapor concentration in kg \( (H_2O) \) m\(^{-3}\), \( W_s \) is the substomatal water
vapor concentration inside the leaf in kg (H\textsubscript{2}O) m\textsuperscript{-3}, and \( \Sigma r \) is the sum of all the water vapor diffusion resistances in s m\textsuperscript{-1} from the atmosphere to the substomatal cavity (normally, the resistance terms considered are \( r_a \), atmospheric boundary layer diffusion resistance; and \( r_s \), leaf stomatal diffusion resistance). The mean rate of leaf assimilation (net or apparent photosynthesis) is given as

\[
A = (C_a - C_s) (\Sigma r')^{-1}
\]

where \( A \) is the CO\textsubscript{2} assimilation rate in kg (CO\textsubscript{2}) m\textsuperscript{-2} s\textsuperscript{-1}, \( C_a \) is the atmospheric CO\textsubscript{2} concentration in kg m\textsuperscript{-3}, \( C_s \) is the substomatal CO\textsubscript{2} concentration (compensation point) in kg m\textsuperscript{-3}, and \( \Sigma r' \) is the sum of all the diffusion resistances in s m\textsuperscript{-1} for CO\textsubscript{2} from the atmosphere to the substomatal cavity inside the leaf (normally, the resistance terms considered are \( r'_a \), the atmospheric diffusion resistance; \( r'_s \), leaf diffusion resistance; and \( r'_m \), cell diffusion resistance).

Penman and Schofield (1951) were the first to analytically examine these relationships based mainly on diffusion approaches and using resistances based on "unstressed" or "potential" crop conditions. They realized similarities in the equations (Eq. [6] and [7]) and analyzed the ratio of the two processes—transpiration ratio in terms of carbohydrate, CH\textsubscript{2}O, and water—with several assumptions for the resistance terms and proposed the following equation:

\[
TA^{-1} = 1.18 \times 10^6 (e^*_s - e) P_b^{-1} (\rho - \rho_s)^{-1}
\]

where \( e^*_s \) is the saturated vapor pressure at the leaf temperature in kPa, \( e \) is the atmospheric vapor pressure in kPa, \( P_b \) is the atmospheric pressure in kPa, \( \rho \) is the atmospheric CO\textsubscript{2} concentration in mg (CO\textsubscript{2}) kg\textsuperscript{-1}, and \( \rho_s \) is the substomatal CO\textsubscript{2} concentration in mg (CO\textsubscript{2}) kg\textsuperscript{-1}. They estimated the transpiration ratio for sugarbeets (\textit{Beta vulgaris} L.) to be 25 when \( \rho_s \) was assumed to be 0 and using the mean atmospheric vapor pressure deficit \((e^*_s - e)\), where \( e^*_s \) is the saturated vapor pressure at air temperature in kPa) as 0.667 kPa for summertime conditions in Great Britain. Since their estimate of the transpiration ratio was about seven times too small as compared to measured values, they concluded that the internal CO\textsubscript{2} concentration must be larger than 0. The transpiration ratio equation proposed by Penman and Schofield (1951), in which the transpiration ratio is directly proportional to the vapor pressure deficit, agreed with the experimental studies conducted by Kisselbach (1916) using container studies in greenhouses maintained at several humidities.

De Wit (1958), using similar arguments, postulated that the transpiration ratio should be linearly related to free water evaporation (potential evaporation) at high solar irradiance levels, almost constant at intermediate solar irradiance levels, and increased as solar irradiance declines to lower levels. Figure 14-1 illustrates his conceptual relationships between the transpiration ratio \((TA^{-1})\), transpiration \((T)\), and assimilation \((A)\) as influenced by the
solar irradiance \( (R) \) or potential evaporation \( (E_o) \) (Penman, 1948). De Wit proposed that the transpiration ratio would not be greatly affected by temperature, water deficits, or mutual shading.

Following the logic of Penman and Schofield (1951), Bierhuizen and Slatyer (1965) used improved concepts of plant resistances to \( \text{H}_2\text{O} \) vapor flux and net \( \text{CO}_2 \) flux to estimate the transpiration ratio as

\[
T A^{-1} = 6.0 \times 10^5 \left( e_s^* - e \right) \left( \Sigma' r' \right) P_b^{-1} \left( \rho - \rho_s \right)^{-1} \left( \Sigma r \right)^{-1}. \tag{9}
\]

If the ratio of diffusion resistances for \( \text{CO}_2 \) to \( \text{H}_2\text{O} \) from the atmosphere to the inside of the leaf is about 2, then Eq. [9] is equivalent to Eq. [8] (Penman and Schofield's). Bierhuizen and Slatyer found that the ratio of these resistances for well-watered cotton (\( \text{Gossypium hirsutum} \) L.) leaves varied from 2 to 8 and depended on both ventilation and irradiance. They reported that Eq. [9] represented a wide range of conditions for different irradiances, ventilation, atmospheric air temperature and humidities, and atmospheric \( \text{CO}_2 \) concentrations.

These relationships for transpiration ratio indicate that the three main factors are: (i) the vapor pressure gradient from the leaf to the air, (ii) the \( \text{CO}_2 \) gradient from the atmosphere to the leaf, and (iii) the diffusion resistances for both \( \text{CO}_2 \) and water. The first factor is mainly an atmospherically controlled variable, although the surface temperature of the leaf will actively respond to atmospheric parameters (e.g., mainly radiation and vapor pressure deficit). The last two factors are clearly related to plant-controlled parameters. These parameters are both genetically determined and environmentally responsive.

Gifford (1974) described the main photosynthetic differences between \( \text{C}_3 \) and \( \text{C}_4 \) plants as: (i) in the \( \text{C}_4 \) photosynthetic pathway, the primary carboxylating enzyme has about twice the affinity for \( \text{CO}_2 \) as in the \( \text{C}_3 \) photosynthetic pathway; (ii) \( \text{C}_3 \) plants have photorespiration (respiration which

![Diagram](Adapted from de Wit (1958))

Fig. 14-1. Conceptual relationships between net assimilation (A), transpiration (T), and transpiration ratio \( (T A^{-1}) \) for leaves or plants and radiation (R) or potential evaporation \( (E_o) \) (de Wit, 1958).
occurs simultaneously with photosynthesis in the light) which requires O₂, 
while this process does not occur in C₄ plants; and (iii) leaves of C₄ type 
plants maintain about one-half the intercellular CO₂ level compared to C₃ 
plants. Downton (1975) and Raghavendra and Das (1978) provided lists of 
C₄ photosynthetic pathway species. Since the assimilation rate will generally 
be larger in C₄ plants due to the higher affinity for CO₂ of the carboxy-
lyating enzyme [this is mainly true in higher light intensities (Goudriaan & 
van Laar, 1978)], the transpiration ratio of C₃ plants will be greater than 
the transpiration ratio of C₄ plants.

Raschke (1975) proposed that the stomatal control by plants when water 
was not limiting could be characterized as: (i) regulating when the internal 
CO₂ concentration is kept within narrow limits and (ii) nonregulating when 
the internal CO₂ concentration is not controlled by the plant. Goudriaan and 
van Laar (1978) demonstrated examples of both situations. Van Keulen and 
van Laar (1986), using the model described in de Wit (1978), computed values 
of the transpiration ratio for C₃ and C₄ crops of both stomatal regulation 
types (R—regulating and NR—nonregulating) for three atmospheric humidity 
levels (Table 14–1). These estimates demonstrate the range of transpiration 
ratios that might be found and the complex nature of the relationship 
between assimilation and transpiration.

The close coupling between photosynthesis and transpiration is obvi-
ous since CO₂ and H₂O simultaneously move through the stomata. The 
diffusive conductance of the stomata opening imposes a major control on 
the rates of both processes, although the internal CO₂ concentration and the 
external H₂O vapor concentration determine the magnitude of the respecti-
tive gradients. However, changes in stomatal resistance may not necessarily 
affect transpiration and assimilation similarly (Cowan & Troughton, 1971).

Cowan (1977) and Cowan and Farquhar (1977) proposed that plants 
dynamically adjust their stomatal resistance to maintain an optimum balance 
between assimilation and transpiration. For a given daily transpiration rate, 
the resistance adjusts to provide the maximum daily assimilation, and for 
a given daily assimilation rate, the daily transpiration rate is minimized. This 
concept was clarified in subsequent work (Cowan, 1982; Schulze & Hall, 1982) 
and defined such that

\[ \frac{\partial T}{\partial A} = \lambda \]  

[10]
Fig. 14–2. Hypothetical combinations of the average rate of assimilation ($\bar{A}$) and transpiration ($\bar{T}$) per unit leaf area per unit time of a plant during a day. Each point on the broken curve corresponds to a particular constant stomatal resistance. Each point on the full curve corresponds to a particular variation of stomatal resistance that is optimal, in the sense that no other variation could lead to a smaller $\bar{T}$ at the same time $\bar{A}$, or a larger $\bar{A}$ at the same $\bar{T}$ (Cowan, 1982).

where $\lambda$ is a constant Lagrange multiplier. Figure 14–2 illustrates this concept for daily assimilation ($\bar{A}$) and daily transpiration ($\bar{T}$) from Cowan (1982). This figure shows the daily temporally and spatially averaged (spatial averaging is over the foliage surface) values of CO$_2$ assimilation ($\bar{A}$) and transpiration ($\bar{T}$) where the particular day represents a single point on the curve. Cowan (1982) stated that “each point on the curve (Fig. 14–2) that bounds this region represents a unique variation of stomatal aperture that could not have been bettered—in the sense that no other variation could have led to a smaller $\bar{T}$ at the same $\bar{A}$, or a larger $\bar{A}$ at the same $\bar{T}$.” This optimal stomatal control theory has been verified from experimental results by Farquhar et al. (1980) over a range in ambient temperatures and humidities, by Hall and Schulze (1980) in the laboratory, and by Field et al. (1982) with field data. However, Cowan (1986) stated that “the paradigm of optimality can be no more than an approximation to the truth.” Farquhar and Sharkey (1982) discussed that this theory requires both feedback control for the internal CO$_2$ concentration and feedforward control for humidity and radiation influences that require a close correspondence between assimilation and conductance (Wong et al., 1979).

The relationship between assimilation and transpiration is well founded because of the coupling between CO$_2$ influx and water efflux from the leaf. Stanhill (1986) reviewed many methods to decouple transpiration and photosynthesis in order to decrease the transpiration ratio. He discussed several means to reduce transpiration while maintaining photosynthesis, e.g., increased cuticular and boundary layer resistances, chemical antitranspirants, plant breeding (Crassulacean acid metabolism [CAM] pathways), selective spectral modification of radiation, as well as means to increase dry matter
production while not increasing transpiration, e.g., plant breeding (C₄ and CAM metabolism pathways compared to C₃ pathways). Clearly, the transpiration ratio is associated with the plant species, the plant and atmospheric diffusion resistances for both CO₂ and water, and various environmental parameters (most notable are solar radiation, air temperature, and vapor pressure deficit).

B. Dry Matter-Transpiration Relationships

The interpretation of the relation between dry matter production and assimilation is difficult since aboveground dry matter is usually measured and not total dry matter. This inconsistency can lead to some incorrect conclusions and requires close attention to how various components of water use efficiency are expressed (Sinclair et al., 1984). Fischer and Turner (1978) summarized many reports of the similarities between the transpiration ratio within C metabolism pathways. They reported transpiration ratios of 667, 303, and 50 kg of water per kilogram of dry matter for C₃, C₄, and CAM plant species, respectively. Stanhill (1986) grouped the transpiration ratios from the container studies from Akron, CO (Shantz & Piemeisel, 1927), by CO₂ metabolism groups. He reported that the 51 C₃ species had transpiration ratios of 640 ± 165 kg of water per kilogram dry matter, and the 14 C₄ species had transpiration ratios of 320 ± 43 kg of water per kilogram dry matter.

Briggs and Shantz (1913a) investigated the effect of the environmental conditions on durum wheat (Triticum turgidum L. var. durum) and sorghum [Sorghum bicolor (L.) Moench] dry matter production and transpiration in Akron and Dalhart, TX. They reported that transpiration ratio for wheat was approximately proportional to pan evaporation for the growing seasons for Akron and Dalhart but that the transpiration ratio for sorghum was nearly constant (1–5% increase at Dalhart compared to Akron for 2 yr) at both locations in spite of a 10 to 14% higher pan evaporation at Dalhart compared to Akron for the 2 yr (1910 and 1911).

De Wit (1958) reanalyzed the early container experiments and found that crop dry matter production was linearly related to the ratio of transpiration to pan evaporation (a sunken pan 1.83 m diam., 0.61 m deep, with the water level maintained at soil level, later called a "BPI [Bureau of Plant Industry] sunken pan") for climates with bright growing season sunshine. De Wit expressed the relationship as

\[ P = m_e T E_e^{-1} \]  \[ 11 \]

where \( P \) is the crop dry matter production in kg container⁻¹, \( m_e \) is a crop specific proportionality coefficient in mm d⁻¹ (the subscript e on \( m \) and \( E \) refer to the data recorded within screened plot enclosures), \( T \) is the transpiration in kg container⁻¹, and \( E_e \) is the pan evaporation in mm d⁻¹ averaged over the growing period. De Wit computed values of \( m \) for sorghum, wheat (Triticum durum Desf.), and alfalfa (Medicago sativa L.) from data
of Briggs and Shantz (1913a, 1914), Shantz and Piemiesel (1927), and Dillman (1931) which were 0.0252, 0.0139, and 0.00662 mm d\(^{-1}\), respectively. Figure 14-3 illustrates the relationships for these three crops between the container production and the ratio of the container transpiration to BPI pan evaporation as reported by de Wit (1958). De Wit reported that the standard errors of the lines through the origin were 0.025, 0.015, and 0.020 kg for the sorghum, wheat, and alfalfa, respectively. He concluded that the relationship (Eq. [11]) was accurate except when the production was small. For conditions outside of the screened plots, de Wit used a relation between transpiration inside and outside the screened plots and a relation between BPI pan evaporation and \(E_o\) to adjust the \(m\) values to 0.0207, 0.0115, and 0.0055 mm d\(^{-1}\) for the sorghum, wheat, and alfalfa, respectively. He found that similar cultivars within several species from experiments and environments had similar values of \(m\); and he concluded that for a first approximation, one could assume that \(m\) was a constant, depending only on the crop species. However, de Wit found that in several experiments in the Netherlands (he noted that the precision of these experiments for this purpose was not as good as those conducted earlier in the USA), production was more accurately estimated by the equation

\[
P = n \, T
\]

where \(n\) is a crop specific coefficient of proportionality in units of kg kg\(^{-1}\). He determined that the value of \(n\) from the experiments in the more temperate climates was indeed different from \(m \, E_o^{-1}\). De Wit analyzed several fertility experiments conducted in containers and determined that \(m\) and \(n\) were reduced when production was seriously limited by nutrient availability. He proposed that \(m\) and \(n\) should be independent of nutrient status if the production was mainly limited by other factors. De Wit found that the value of \(n\) was consistent in many experiments in which the crops were allowed to

![Fig. 14-3. Relationships between production and the ratio of transpiration to pan evaporation for sorghum, wheat, and alfalfa (de Wit, 1958).](image-url)
deplete various amounts of soil water so long as the containers were not overly irrigated (aeration problems) or greatly underirrigated (greatly stressed). He also extrapolated his methods to analyze the relationship between production and irrigation water application, albeit de Wit acknowledged the many pitfalls of this step.

Arkley (1963) postulated that crop growth and transpiration were related but that advection would distort the relationship. He reanalyzed the data summarized largely by de Wit (1958) and determined that crop production, when fertility was constant or adequate as estimated by Eq. [11] and [12], could be unified with the equation

$$P = k_a T (100 - H)^{-1}$$  \[13\]

where $k_a$ is a crop specific coefficient in units of percentage, and $H$ is the mean daily relative atmospheric humidity in percentage during the growing season. Equation [13], thus, effectively provided a means to use the relationships presented by de Wit (1958) in various climatic conditions (mainly advection differences). Arkley estimated that daytime atmospheric relative humidity values should be more meaningful in Eq. [13] than daily averaged atmospheric relative humidity values. He also investigated the relationship between crop dry matter production and the ratio of transpiration to vapor pressure deficit arranged in the following form:

$$P = k'_a T (e^* - e)^{-1}$$  \[14\]

where $k'_a$ is the crop specific coefficient in units of kPa. Since $H = 100 e (e^*)^{-1}$, the crop coefficients are related by the following:

$$k'_a = 0.01 e^* k_a.$$  \[15\]

Since $e^*$ is temperature dependent, the crop specific coefficient, $k'_a$, would be temperature dependent also. Arkley recommended that Eq. [13] contained the necessary temperature dependency within the $H$ term. This contradicted the theory proposed by Penman and Schofield (1951) that potential assimilation was inversely proportional to atmospheric vapor pressure deficit ($e^* - e$).

Hanks et al. (1969) reported that dry matter was linearly related to evapotranspiration for wheat (T. aestivum L.), millet (Panicum miliaceum L.), oat (Avena sativa L.), and grain sorghum at Akron, CO, in both lysimeters and field plots. They estimated soil evaporation by several techniques and subtracted it from total evapotranspiration to obtain an estimate of transpiration. They reported $m$ values (Eq. [11]) of 125 kg ha$^{-1}$ d$^{-1}$ for winter wheat [note that these units are derived for $P$ in units of kg ha$^{-1}$, $T$ in units of mm, and $E_o$ in units of mm d$^{-1}$], 94 to 223 kg ha$^{-1}$ d$^{-1}$ for oat, 132 to 167 kg ha$^{-1}$ d$^{-1}$ for millet, and 141 kg ha$^{-1}$ d$^{-1}$ for grain sorghum. Hanks (1974) concluded that for studying only the effects of limited water on crop production, the de Wit (1958) or Arkley (1963) relationships
(Eq. [11] and [13]) could be simplified since \( m \) and \( E_o \) would be constant for a given crop in a given year to the following:

\[
P P_m^{-1} = T T_m^{-1}
\]

where \( P_m \) is maximum or potential dry matter production in kg ha\(^{-1}\), \( T \) is transpiration for the growing season in mm, and \( T_m \) is the maximum or potential transpiration when soil water does not limit transpiration or yield. Hanks (1974) demonstrated validation of Eq. [16] for yield prediction with model estimates of transpiration for corn (\textit{Zea mays L.}) and grain sorghum.

Tanner and Sinclair (1983) researched the relationship developed by Bierhuizen and Slayyer (1965) (Eq. [9]) in order to determine if current simplified relationships for transpiration and assimilation would lead to an expression for the transpiration ratio of crops that would be consistent with observed differences in the transpiration ratio among species. They developed an equation for transpiration ratio from a crop which is

\[
T P^{-1} = 1.5 \times 10^4 \rho_a \epsilon L_T B' (e^* - e) (a b c P_b C_a \text{LAI}_D)^{-1}
\]

where \( T \) is in mm d\(^{-1}\), \( P \) is in kg ha\(^{-1}\) d\(^{-1}\), \( a \) is the molecular weight of hexose to carbon dioxide (CH\(_2\)O CO\(_2\)^{-1}, 0.68), \( b \) is a factor for the conversion of CH\(_2\)O to biomass which ranges from 0.33 to 0.83 (Penning de Vries, 1975) \( c \) is the CO\(_2\) factor \([\rho - \rho_s] \rho^{-1}\), where \( \rho_s \) is the intercellular CO\(_2\) concentration in the leaf in mg kg\(^{-1}\) and \( \rho \) is the atmospheric CO\(_2\) concentration in mg kg\(^{-1}\) which is approximately constant for a crop with values of 0.3 for C\(_3\) crops and 0.7 for C\(_4\) crops (Wong et al., 1979), \( C_a \) is the atmospheric CO\(_2\) density in kg m\(^{-3}\), \( \text{LAI}_D \) is the leaf area index of leaves directly exposed to incident radiation, \( \rho_a \) is the air density in kg m\(^{-3}\), \( \epsilon \) is the molecular weight of water vapor to air (0.622), \( L_T \) is the effective transpiration leaf area index, and \( B' \) is a correction term for the shaded and non-shaded leaf area which is approximately \( 1 \pm 0.2 \) when \( \text{LAI} > 3 \). They proposed that dry matter production could be estimated by

\[
P = 1.0 \times 10^4 \int [k_d T (e^* - e)^{-1}] \, dt
\]

where dt is days and \( k_d \) is a crop-specific coefficient determined as

\[
k_d = (0.667 a b c P_b C_a \text{LAI}_D) (\rho_a \epsilon L_T B')^{-1}
\]

where \( k_d \) is in kPa. They reported consistent agreements between the predicted value of \( k_d \) and experimentally derived values (Table 14-2) from several experiments where the necessary data were measured, except for potato (\textit{Solanum tuberosum L.}). Tanner (1981) discussed this difference in detail, but did not find the source for the difference. Tanner and Sinclair (1983) emphasized the following points in their review of the use of Eq. [18]:

1. Since many of the factors in Eq. [18] and [19] are correlated, great care is necessary in applying the equation to experimental data. In particu-
Table 14–2. Comparison of computed estimates for \( k_d \) by Eq. [19] and experimental measurements (from Tanner & Sinclair, 1983).

<table>
<thead>
<tr>
<th>Crop</th>
<th>Location</th>
<th>Source</th>
<th>Experimental ( k_d )</th>
<th>Computed ( k_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>Davis, GA</td>
<td>Stewart et al., 1977</td>
<td>0.0100</td>
<td>0.0118</td>
</tr>
<tr>
<td>Sorghum</td>
<td>Manhattan, KS</td>
<td>Teare et al., 1973</td>
<td>0.0138</td>
<td>0.0118</td>
</tr>
<tr>
<td>Potato</td>
<td>Madison, WI</td>
<td>Tanner, 1981</td>
<td>0.0065</td>
<td>0.0055</td>
</tr>
<tr>
<td>Potato</td>
<td>Netherlands</td>
<td>Rijtema &amp; Endrödi, 1970</td>
<td>0.0015</td>
<td>0.0055</td>
</tr>
<tr>
<td>Alfalfa</td>
<td>Madison, WI</td>
<td>Tanner &amp; Sinclair, 1983</td>
<td>0.0043</td>
<td>0.0050</td>
</tr>
<tr>
<td>Soybean</td>
<td>Manhattan, KS</td>
<td>Teare et al., 1973</td>
<td>0.0040</td>
<td>0.0041</td>
</tr>
</tbody>
</table>

lar, they emphasized that \( \int T[B' (e^* - e)^{-1}] \) dt would not equal \( T_T \) \( [B' (e^* - e)^{-1}] \) where \( T_T \) is the season total transpiration in mm and equal to \( \int T \) dt.

2. The partitioning of sunlit and shaded leaves is necessary to estimate the effects of the environment on leaf temperature to improve the equation. Leaf temperature in low air temperature environments might be warmer than air, while sunlit leaf temperatures in warmer temperature environments might be cooler than the surrounding air.

3. Since the daytime values of transpiration and assimilation should be the most important, the daytime vapor pressure deficit would be more sensitive than the daily mean vapor pressure deficit.

4. Improvements in the transpiration ratio through breeding must result in modifications to the \( c \) factor in Eq. [17] where \( c = [1 - (\rho_s \rho^{-1})] \). They cited as support for their position the similarity of \( k_d \) values for corn from 1912 to 1975, although they recognized that partitioning of dry matter into grain has improved through breeding.

Although Tanner and Sinclair (1983) indicated that the potential to increase \( k_d \) through breeding was limited, Farquhar and Richards (1984), demonstrated a screening technique that showed differences in \( \rho_s \) among wheat genotypes. Richards (1987) reviewed the potential to use the differences in the transpiration ratio (or \( k_d \)) in breeding programs.

The crop dry matter production relationship to transpiration is quantitatively similar to the relationships between assimilation and transpiration. The main factors affecting the relationship are the \( \text{CO}_2 \) metabolism pathway and environmental factors (e.g., vapor pressure deficit, potential evaporation, and air and leaf temperature). The crop and atmospheric diffusion resistances that affect both photosynthesis and transpiration are also important but are more difficult to quantify at the crop level.

C. Economic Yield-Evapotranspiration Relationship

The previous discussions have considered only assimilation and dry matter production. Economic production is normally only a portion of the total dry matter production of a crop. In many cases, the quality of this portion
of the crop production significantly affects its economic value. Transpiration, likewise, is a portion of the total water supply provided to produce a specific crop. Additionally, transpiration is practically impossible to measure on a field level, and even evapotranspiration is sometimes difficult to measure. Therefore, many times economic evaluation of irrigation systems or irrigation management is made on the basis of economic yield and applied water. The ratio of crop yield (economic yield) to applied irrigation water has often been termed water use efficiency, but this term is confusingly applied and does not always correctly express how applied irrigation water impacts crop productivity.

1. Economic Yield-Dry Matter Production Relationships

The economic product of a crop can be the dry matter as in forages; but, more likely, it is either the grain, fiber, seed, fruit, root, tuber, or some other plant component. Since the economic product is included in the total plant dry matter, it is logical to quantify the partitioning of the economic production in terms of the total crop or plant dry matter. Many experiments are not concerned with the dry matter production, only the economic production; therefore, the partitioning between economic yield and total dry matter yield is not often reported. Even when dry matter production is determined, the root component of the dry matter yield is rarely measured. The ratio of economic yield to aboveground dry matter yield is termed the harvest index (Donald & Hamblin, 1976) and is useful in characterizing a wide range of agronomic experiments. The harvest index is defined as

\[ H_i = Y \left( P_s + Y \right)^{-1} \]  \hspace{1cm} [20]

where \( H_i \) is the harvest index (dimensionless), \( Y \) is the economic yield (dry basis) in kg ha\(^{-1}\), and \( P_s \) is the stover yield in kg ha\(^{-1}\). Defining aboveground dry matter (\( P_a \)) as the sum of \( P_s \) and \( Y \) and the total crop dry matter production (\( P_t \)) as sum of \( P_a \) and the root dry matter production (\( P_r \)), relationships between economic yield and dry matter yield can be expressed as follows:

\[ Y = H_i \ P_a \]  \hspace{1cm} [21a]

and

\[ Y = H_i \ (P_t - P_r) \]  \hspace{1cm} [21b]

where \( P_a \) is aboveground dry matter in kg ha\(^{-1}\), \( P_t \) is total dry matter production in kg ha\(^{-1}\) (\( P_t = P_s + Y + P_r \)), and \( P_r \) is root dry matter production in kg ha\(^{-1}\). Since \( P_a \) includes \( Y \), it is evident that Eq. [21a] should be indicative of the high degree of self correlation that must exist between \( Y \) and \( P_a \).

De Wit (1958) indicated that the slope of the linear relationship between grain yield and aboveground dry matter yield of Kubanka wheat (\( T. \ turgidum \) L. var. \( durum \)) from several container studies in the USA was 0.36 when
the relationship was forced through the origin. De Wit (1958) also reported that the relationship between grain yield of Kubanka wheat and aboveground dry matter yield from field studies of the early USDA dryland research sites in the Great Plains of the USA (Cole & Mathews, 1923) was linear with a slope of 0.42 and passed through the origin, assuming that 15% of the dry matter was left in the field as stubble. The linear regression between the reported grain yield (assuming 0.773 kg L\(^{-1}\) for wheat) and total reported dry matter yield (total of grain and straw yields) from Cole and Mathews' data, as shown in Fig. 14–4, was

\[
Y = 0.404 \left( P_s + Y \right) - 0.178 \quad (r^2 = 0.876, N = 83)
\]

[22]

where \(Y\) and \(P_s\) are expressed in units of Mg ha\(^{-1}\). The intercept \((-0.178\text{ Mg ha}\(^{-1}\))\) was significantly different from zero \((P < 0.05)\) and the slope was 0.382 when the relationship was forced through the origin. This relationship is remarkably similar to one determined by Aase and Siddoway (1981) for spring wheat where the intercept was \(-0.298\) and the slope was 0.478 \((r^2 = 0.946)\) based on data from 20 different experiments. The following statement by Cole and Mathews (1923) seems appropriate to describe this relationship:

... when affected by drought the wheat crop seems to spend its last energy in producing grain, and that if there is any chance at all, it will produce some yield of grain. This study indicates that a high yield of straw means a high yield of grain. There have been a few cases when exceptionally favorable weather enables wheat to fill so well that the yield of grain was out of proportion to the yield of straw. These years are very infrequent, and as a whole the yield of grain and straw are nearly proportional.

Speath et al. (1984) reported that the harvest index of soybean \([Glycine max\text{ L. (Merr.)}]\) was a conservative characteristic within specific cultivars. Snyder and Carlson (1984) reviewed the harvest index in relation to improved

\[GY = 0.404 \text{ DM} - 0.178\]

\[r^2 = 0.876, N = 83\]

Fig. 14-4. Relationship between grain yield and aboveground dry matter yield of Kubanka wheat from dryland field studies in the Great Plains of the USA with data from Cole and Mathews (1923).
economic yields of crops through plant breeding and also discussed both environmental and biological factors that might affect the harvest index.

Slabbers et al. (1979) investigated the relationship between grain yield (economic yield) and dry matter yields for grain sorghum and corn. They reported the following linear regression equations:

\[
\text{Sorghum: } Y = 0.58 \left( P_5 + Y \right) - 1.26 \quad (r^2 = 0.941) \quad [23]
\]

\[
\text{Corn: } Y = 0.49 \left( P_5 + Y \right) - 1.21 \quad (r^2 = 0.865) \quad [24]
\]

where \( Y \) and \( P_5 \) are in units of Mg ha\(^{-1}\) (note that it was not explicitly stated whether the grain was dry or at standard water content). These equations accounted for 94 and 86% of the variation in yield of sorghum and corn, respectively, when tested against independent data. Figure 14-5 illustrates the relationship described by Eq. [23] for sorghum and the harvest index \( [Y (P_5 + Y)^{-1}] \). Figure 14-5 demonstrates the importance of the intercept in the relationship between economic yield and aboveground dry matter yield of crops. The economic yield would be better estimated by an equation that accounts for the dry matter yield threshold. At the higher levels of dry matter yields, the harvest index become conservative, as shown in Fig. 14-5.

The relationship between economic yield and dry matter yield has been widely used in many procedures to estimate the effects of crop water use on crop economic yield (e.g., Slabbers et al., 1979; Doorenbos & Kassam, 1979; van Keulen & Wolf, 1986). Generally, the relationship between economic yield and dry matter yield is based on a concept utilizing the harvest index as a constant (Eq. [21a]). The adjusted harvest index \( (H_{ia}) \) defined as

\[
H_{ia} = Y \left( P_5 + Y - P_r - P_o \right)^{-1}
\]

[25]

![Graph](image)

**Fig. 14-5.** Relationship between grain yield, harvest index, and aboveground dry matter yield of grain sorghum based on the regression equation presented by Slabbers et al. (1979).
where $P_0$ is the dry matter yield threshold (amount of total stover production necessary to produce the first increment of economic yield) could improve the accuracy of the relationship between economic yield and dry matter yield. If $P_r$ is neglected and the above examples are used, $P_0$ for wheat, sorghum, and corn could be estimated as 442, 2178, and 2465 kg ha$^{-1}$, respectively, with the resulting values of 0.40, 0.58, and 0.49 for the adjusted harvest indices, respectively. The relationship between economic yield and dry matter yield might be more accurately determined if the partitioning components of economic yield, such as seed number, seed mass, etc., could be estimated.

2. Transpiration-Evapotranspiration Relationships

The measurement of transpiration in the field is complex (Klocke et al., 1985) and subject to many errors. Even the field measurement of evapotranspiration can be rather complex and difficult in many situations where the drainage from the root zone, water uptake from saturated zones, and runon and runoff from the area are difficult to measure, both temporally and spatially. Generally, soil water balance techniques are used to measure seasonal evapotranspiration from crops. Precise field soil water balance measurements are usually possible only when using lysimeters to precisely define the water movement across the lower soil boundary. Several micrometeorological techniques can be used to measure the energy balance and evapotranspiration from crops as described in other chapters in this Monograph.

Most often, the transpiration is estimated from evapotranspiration measurements using (i) subtraction of an estimate of $E_s$ (usually, $E_s$ is assumed to be the intercept of the P-ET linear regression), which is most often taken to be a seasonal constant from the measured seasonal ET (Hanks et al., 1969); (ii) daily water balance simulation using empirical functions to separately calculate $T$ from daily calculations (or measurements) of ET using measured plant parameters such as leaf area index or ground cover (Ritchie, 1972; Tanner, 1981; Howell et al., 1984; Hanks, 1985); or (iii) measuring $E_s$ and subtracting it from measurements of ET (Lascano et al., 1987). All of these measurement techniques yield indirect estimates of transpiration. Direct plant measurements of water movement rates have been made using the heat-pulse velocity technique (Bloodworth et al., 1955), but the estimation of transpiration flux remains difficult because of volume calibration difficulties as well as sampling limitations. However, newer techniques and improvements in heat-pulse instrumentation appear to greatly solve calibration problems (Sakuratani, 1984) or even eliminate them (Baker & van Bavel, 1987).

The relationship first proposed by Ritchie and Burnett (1971) or variations (Tanner & Jury, 1976; Kanemasu et al., 1976; Al-Khafaf et al., 1978) have been widely used to estimate field transpiration of "unstressed" crops. The Ritchie and Burnett relationship developed for cotton and grain sorghum is

\[
T = E_o \left[ -0.21 + 0.70 \left( \text{LAI} \right)^{1/2} \right]
\]  

[26]
where $T$ is transpiration in mm d$^{-1}$, LAI is the leaf area index, and $E_o$ is "potential" evapotranspiration in mm d$^{-1}$. It is interesting to note that the data from Ritchie and Burnett (1971) illustrate that the ratio of transpiration to "potential" evapotranspiration ($T/L/E_o$) was more closely related to aboveground dry matter than to either leaf area index or ground cover for cotton and grain sorghum and two row spacings of grain sorghum. The data for $T/L/E_o$ and LAI for Ritchie and Burnett (1971) can be closely approximated by the simpler equation

$$T = E_o [1 - \exp(-0.8 \text{ LAI})]$$  \hspace{1cm} [27]

where $\exp$ represents the exponential function [$\exp(x) = e^x$] with little loss in accuracy. Ritchie (1983) discussed the bias in Eq. [26] and [27] due to the advective influences that enhance transpiration with dry soil surface conditions. He presented a curve for wet soil surface conditions that would suggest that the exponential coefficient in Eq. [27] ($-0.8$) would be reduced to about $-0.38$ to $-0.40$ when the soil surface was wet.

The effects of reduced soil water contents (or, in effect, soil water potential) on transpiration are more difficult to precisely estimate. Transpiration under soil water deficits is strongly influenced by the crop rooting depths, rooting densities, soil hydraulic properties, and the evaporative demand. Campbell and Campbell (1982) illustrated the influences of rooting density, soil water potential, and evaporative demand on crop water uptake from the soil by using the Ohms-law electrical analogy to simulate water flow from the soil through the plant to the atmosphere.

The relationship between transpiration and evapotranspiration is not clearly defined in most cases. Various model forms have been used to estimate transpiration from "potential" or "maximum" evapotranspiration estimates. These relationships are often site as well as crop specific.

3. Economic Yield-Evapotranspiration Relationships

All of the previous discussion illustrate the estimation of dry matter by using transpiration as the independent variable. The estimation of transpiration from the total evapotranspiration is difficult (Hanks & Rasmussen, 1982). Since total evapotranspiration (ET) is the process most closely related to transpiration that can be measured in the field, many approaches have been based on the economic yield relationship to ET.

Cole and Mathews (1923) and Mathews and Brown (1938) investigated grain yield for winter wheat and sorghum across the southern Great Plains in the USA in relation to precipitation, the practice of fallowing, and effects of growing conditions (soils and locations). They used linear regression techniques to evaluate the function

$$Y = b \text{ ET} + a$$  \hspace{1cm} [28]

where $Y$ is grain yield in kg ha$^{-1}$, ET is the estimated growing season evapotranspiration in mm, $a$ and $b$ are regression coefficients in units of kg
ha\(^{-1}\) and kg ha\(^{-1}\) mm\(^{-1}\), respectively. They estimated ET as growing season precipitation plus soil water depletion from seeding until harvest. They found \(a\) to be negative as a result of soil water evaporation (Hanks, 1974) (note that the soil evaporation can be approximated by the ratio \(a b^{-1}\)). The equation determined by Mathews and Brown (1938) for winter wheat at Garden City and Colby, KS, was

\[
Y = 5.19 \text{ ET} - 972 \quad (r^2 = 0.561, N = 81) \quad [29]
\]

where \(Y\) is in kg ha\(^{-1}\) and ET is in mm. The average error in wheat yield estimation over 20 yr at Colby, KS, was 98 kg ha\(^{-1}\). They tested their model with data from three other USDA dryland stations in Texas and Oklahoma. The results indicated that the model explained slightly >50% of the variance in the yield data from the additional three sites. In addition, the model was not biased in that the intercept \((-117\ \text{kg ha}^{-1})\) was not different from zero \((P < 0.05)\) and the slope was not different from 1.0 \((P < 0.05)\). However, the standard error of the model was 430 kg ha\(^{-1}\) when tested against estimated yields at these three sites. By current standards, this model would seem to be quite applicable for the intended purpose of estimating dryland winter wheat production in the southern Great Plains, although the model is rather site specific. The slope from the Mathews and Brown (1938) regression equation \((5.19\ \text{kg ha}^{-1}\ \text{mm}^{-1})\) compares well to 6.38 kg ha\(^{-1}\) mm\(^{-1}\) from a later dryland wheat study at Bushland, TX (Johnson & Davis, 1980). These empirical models of crop production-evapotranspiration are widely used for many agronomic, engineering, and/or economic purposes but are widely criticized for site specificity, effects of specific periods of water stress effects, the lack of climatic influences, and empiricisms that do not increase the understanding of the fundamental relationships between production and water use. Many debates have occurred regarding whether the empirical relationship between economic yield of a crop and crop ET was linear, quadratic, or some other function (Barrett & Skogerboe, 1980).

Many crop production-evapotranspiration models have evolved to predict economic crop yield. Various techniques were used to address the crop yield response in relation to ET and to ET deficits in specific crop growth stages. Jensen (1968) proposed two models of crop yield in relation to ET during specific crop growth stages: (i) for determinate crops and (ii) for indeterminate crops. His model for determinate crops is

\[
Y = \prod_{i=1}^{n} \left[ \text{ET} \ \text{ET}_m^{-1} \right]^{\lambda_i} \quad [30]
\]

where \(Y_m\) is maximum or potential grain yield in kg ha\(^{-1}\) with water not limiting production, \(\text{ET}_m\) is the crop water use in mm with water not limiting production, \(\lambda_i\) is the relative sensitivity factor (dimensionless) of the crop to water deficits in growth stage \(i\), and \(n\) is the number of growth stages. The right side of Eq. [30] is a product. Jensen gave \(\lambda\) values of 0.5, 1.5, and
0.5 for three periods of grain sorghum as emergence to boot, boot to milk stage, and milk to harvest, respectively. His indeterminate crop yield model was of the form

\[ Y Y_m^{-1} = \left[ \sum_{i=1}^{n} \lambda_i (ET)_i \right] \left[ \sum_{i=1}^{n} \lambda_i (ET_m)_i \right]^{-1} \]  \[31\]

Jensen stated that the primary difference between Eq. [30] and [31] was that for indeterminate crops, the effects of water stress on yield during specific growth stages are independent of other growth stages. Stewart et al. (1977) and Doorenbos and Kassam (1979) proposed to estimate crop yield in relation to ET as

\[ Y Y_m^{-1} = \{1 - B[1 - (ET)(ET_m)^{-1}]\} \]  \[32\]

where \( B \) is the yield response factor (dimensionless). Hanks and Rasmussen (1982) determined that soil evaporation and maximum transpiration could be estimated from Eq. [32] as follows:

\[ E = ET_m (1 - B^{-1}) \]  \[33\]

\[ T_m = ET_m B^{-1} \]  \[34\]

where \( E \) is soil water evaporation in mm, and \( T_m \) is maximum transpiration in mm. The value of \( B \) should be >1 if the intercept of the yield and ET line (or curve) is negative. Hanks (1983) discussed that the \( B \) values <1 reported by Doorenbos and Kassam (1979) and Stewart et al. (1977), probably resulted from use of limited data and incorrect estimates of \( ET_m \) and \( Y_m \). Hanks also proposed that the values of \( B \) and \( m \) (de Wit, 1958) were related as

\[ m = Y_m E_o B ET_m^{-1} \]  \[35\]

where \( E_o \) is the mean daily growing season potential ET in mm d\(^{-1}\). Hanks and Rasmussen (1982) and Hanks (1983) reviewed additional models that have been used to relate crop yields to water use which, in general, are basically some variation or combination of the above models. Although the discussion is limited in the literature, the yield response of the crop is considered to be constant at \( ET \geq ET_m \) (meaning that soil evaporation is larger than the minimum \( E \) necessary to produce \( Y_m \)).

To summarize this section, an example analysis might be enlightening and useful in illustrating these concepts. A winter wheat study (Jensen & Sletten, 1965) that has been used in the literature for this purpose (Ritchie, 1983) was chosen. The study consisted of six water treatments and six fertility treatments over 3 yr. Dry matter yields were not reported, but straw-to-grain ratios were reported for selected treatments in 1955–56. Stover yield for the 1955–56 data was computed as the product of the straw-to-grain ratio times the reported grain yields. Total aboveground dry matter production for the 1955–56
data was computed as the sum of the stover and grain yields. The source
of the data did not report the water contents of the grain or the straw, but
the combined water contents are likely <8% (wet basis) and no correction
was applied for water in the estimated dry matter or grain. The ET was mea-
sured by soil moisture sampling, and the plots were level borders so no runoff
occurred. The reported data from the M1 through M6 water treatments and
the F2, F4, and F5 fertility treatments (34 kg ha\(^{-1}\) P\(_2\)O\(_5\) each with 0 kg ha\(^{-1}\),
90 kg ha\(^{-1}\), and 135 kg ha\(^{-1}\) of N, respectively) were utilized in this anal-
ysis since these were the only treatments with published data for the straw-to-
grain ratio. The grain yield was highly correlated to the estimated total
aboveground dry matter as illustrated by the regression equation

\[ Y = 0.388 (P_s + Y) - 0.05 \quad (r^2 = 0.976, N = 18) \quad [36] \]

where \(Y\) and \(P_s\) are in Mg ha\(^{-1}\). Both the estimated aboveground dry mat-
ter yield \((P_s + Y)\) and grain yield \((Y)\) were highly correlated to the mea-
sured ET as expressed by the regression equation

\[ (P_s + Y) = 0.0169 \text{ ET} - 5.00 \quad (r^2 = 0.870, N = 18) \quad [37] \]

\[ Y = 0.00648 \text{ ET} - 1.96 \quad (r^2 = 0.846, N = 18) \quad [38] \]

where \(Y\) and \(P_s\) are in Mg ha\(^{-1}\), and ET is in mm. The relative reduction
in both estimated aboveground dry matter yield and grain yield was also highly
correlated to the relative reduction in ET as expressed by Eq. [32] with coeffi-
cients of determination of 0.878 and 0.711, respectively. Figure 14–6 shows
the relationship between the relative decrease in total dry matter yield in rela-
tion to the relative ET decrease for the 1955–56 season for the six water and
three fertility treatments. Maximum dry matter was 9.10 Mg ha\(^{-1}\), and
\(Y_m\) was 3.37 Mg ha\(^{-1}\), while ET\(_m\) was taken as 864 mm for the 1955–56 sea-

![Graph showing the relationship between the relative decrease in dry matter yield of Concho winter wheat at Bushland, TX, in 1956 (Jensen & Sletten, 1965) and the relative decrease in ET.

Fig. 14-6. Relationship between the relative decrease in dry matter yield of Concho winter wheat at Bushland, TX, in 1956 (Jensen & Sletten, 1965) and the relative decrease in ET.
son. The slopes of these relationships when forced through the origin to determine $B$ values for Eq. [32] were 1.38 and 1.32 for dry matter and grain, respectively. These $B$ values, along with $E_o$ determined from pan evaporation data reported by Jensen and Sletten (1965) and the values of $Y_m$ and $ET_m$, were used to compute $m$ values (de Wit, 1958) which were 111 kg ha$^{-1}$ d$^{-1}$ and 39 kg ha$^{-1}$ d$^{-1}$ for dry matter and grain, respectively. The estimated dry matter $m$ value is close to that value reported by de Wit (1958) for wheat of 115, the range of values for wheat of 110 to 140 reported by Fischer and Turner (1978) (note that their $m$'s include root biomass), and 125 for wheat reported by Hanks et al. (1969). The proportionality factor, $B$, was similar to the seasonal value for wheat of 1.0 to 1.15 reported by Doorenbos and Kassam (1979), although my value is larger. The soil evaporation component of $ET_m$ was approximately 25% based on the value of $B$, and the transpiration component of $ET_m$ was about 75%. Although these data contain some definite trends that illustrate fertility interactions with irrigation (Ritchie, 1983), in general, the data can be adequately represented by functions similar to Eq. [11] and [32] over a relatively wide range in both fertilizer applications and irrigation water management.

The information regarding economic yield of crops and evapotranspiration can be summarized as illustrated in Fig. 14-7. It should be kept in mind that these functional relationships are only applicable to small plots where (i) the soil is relatively uniform, (ii) all water applications (both rainfall and irrigation) are applied uniformly, (iii) severe water deficits during critical crop growth periods are avoided, (iv) salinity (of either the soil or irrigation water) does not limit production, and (v) fertility and cultural management techniques do not limit production. Figure 14-7 is illustrative of the functions in the above discussion for wheat grown in a variety of locations. Aboveground dry matter yields are linearly related to ET from a point which is about 20 to 25% of $ET_m$ up to the point $P_m$, $ET_m$. It would be conceivable that ET could exceed $ET_m$ if the soil surface was kept wet.

![Fig. 14-7. Diagram illustrating concepts of yield-ET relationships for wheat. The open symbols represent dry matter production and the filled symbols represent grain production.](image-url)
from frequent rains or irrigations that increased soil evaporation. Economic yield (in this example, wheat grain yield) would also increase linearly from a point at approximately 25 to 30% of ETm up to the point $Y_mP_m^{-1}, ET_m$. Although Fig. 14-7 is considered a gross simplification, it provides a discussion framework and is, in fact, realistic, as demonstrated by the data illustrated in the figure for wheat from Singh and Malik (1983), Steiner et al. (1985), and Mogensen et al. (1985). Similar diagrams could be developed with the minimum inputs of $P_m$, $ET_m$, $B$ (or $m$ or $k_d$), and $H_i$. Procedures to develop this information are readily found in Doorenbos and Kassam (1979) and van Keulen and Wolf (1986). Such diagrams can provide production envelopes that can represent the upper limits to expected production. Specific yields might not equal the estimated production limits because of various limitations such as disease, pests, fertility, critical period water stresses, salinity, nonuniform irrigation applications, etc.

II. EFFECTS OF OTHER ENVIRONMENTAL AND MANAGEMENT FACTORS

The previous section has summarized concepts used to describe the relationship between crop assimilation-transpiration, crop dry matter production-transpiration, crop economic production-dry matter production, crop economic production-evapotranspiration. This section will discuss the effects of other factors, i.e., evaporative demand, fertility, salinity, critical periods of water deficits, soil variability, and irrigation application uniformity.

A. Evaporative Demand Effects

Evaporative demand clearly affects the relationships between assimilation-transpiration, crop dry matter-transpiration, and yield-evapotranspiration. The evaporative demand influences are quantified through the vapor pressure deficit in several of the equations presented in the previous section or the environmental factors that influence ETm. Basically, the evaporative demand affects the partitioning between soil evaporation and transpiration which depends on the surface soil wetness and the amount of crop development (Ritchie, 1983) and affects the transpiration ratio directly. Clearly, the addition of extra advective energy to drive the evaporative process will not result in increased crop productivity. Crop dry matter production depends on the amount of photosynthetically active radiation (PAR) absorbed by the crop (Monteith, 1977). Canopy light interception depends on many factors, with the canopy architecture and leaf area index being the important crop factors and the distribution of PAR between direct beam and diffuse components being important radiation parameters. As discussed by Ritchie (1983), both absorbed PAR and ET are driven largely by radiation, but ET is also increased by advective influences (vapor pressure deficit and wind) and nearly maximized at leaf area indices approaching 3, while absorbed PAR continues to increase with leaf area indices exceeding 4 to 5.
B. Fertility Effects

Crop productivity is strongly influenced by nutrition and water availability. Viets (1962) investigated these interactions in terms of water use efficiency for crops with unlimited water supplies. When the water supply to a crop is fixed, any management factor that increases production, such as fertilizers, weed control, disease control, planting density, planting geometry, will increase the water use efficiency. The crop production-evapotranspiration relationships presented in the previous section assume that crop nutrition is adequate and nonlimiting to production.

Crop nutrition through fertilizer applications does not greatly affect crop water use unless significant effects on leaf area development are present. Even in the cases where fertilizers increase leaf area index, generally, the time distribution of the effects results in minor changes in actual crop water use unless the nutrient deficiency is very extreme. Nutrient uptake is largely determined by the nutrient demand to meet sink requirements in the crop materials but can be limited by nutrient status in the soil or by water-limited conditions in soil layers where crop nutrients are available and crop rooting is sufficient for nutrient uptake but where root water uptake is limited by the soil water potential. Crop fertility management can have both positive and negative effects on crop productivity when the water supply is fixed and/or limited (Black, 1966; Viets, 1966). Fertilizer applications to a crop with limited available water could result in early depletion of the limited soil water and the development of severe water deficits during later critical crop development stages, possibly reducing yield and water use efficiency. With sufficient available soil water and a nutrient deficient soil, nutrient additions should increase dry matter and economic yield, thereby increasing the water use efficiency. Jones et al. (1986) reported that neither water nor N stress affected the value of $k_d$ for corn. Viets (1972) concluded that nutrient and water uptake were largely independent processes in crop roots and that plants do not need a constant supply of nutrients.

Rhoads (1984) has summarized the literature dealing with water and N responses of crops which indicates that when N was limiting yield, the water use efficiency was improved sometimes as much as 41\% when higher rates of N were applied. Figure 14–8 illustrates the interactions of N and water applications on aboveground dry matter and grain yield of corn (Stapleinton et al., 1983). Hanks et al. (1983) stated that production surfaces similar to that illustrated in Fig. 14–8 are site specific. The response surfaces shown in Fig. 14–8 illustrate the points: (i) the yield response to irrigation applications will increase with increasing N applications until N no longer limits yield for the amount of irrigation applied; (ii) there is a broad range of N fertilizer applications that result in approximately similar yields (the data in Fig. 14–8 show small dry matter or grain yield increases, regardless of the irrigation level, as N application is increased from 150 to 300 kg ha$^{-1}$); and (iii) N and irrigation applications affect the relationship between economic yield and dry matter yield similarly (no differential effect on harvest index). Thus, these empirical production surfaces are of little general usefulness be-
cause of the site-specific nature of the soil fertilizer interactions, local rainfall patterns, etc.; but they are useful examples of the interaction of irrigation and nutrients.

The relationship between crop yield and nutrient requirement is fully explored by van Keulen (1986a) based on graphical procedures suggested by de Wit (1953). Figure 14–9 illustrates this graphical analysis procedure using an irrigated corn fertilizer uptake study (Stapleton et al., 1983). Both dry

![Graphical representation of crop yield vs. nutrient application and water application]

Fig. 14–8. Response surfaces of corn dry matter and grain yield (dry) to irrigation applications and rainfall and to N applications. Data from Stapleton et al. (1983).
matter yield and grain were affected by the applied water and N fertilizer (Fig. 14-8). Part A of Fig. 14-9 shows the corn grain yield as affected by the fertilizer and four irrigation levels, W1 through W4. Part D of Fig. 14-9 shows the efficiency of the N recovery by the corn crop as affected by the different water levels. The high N recovery, even exceeding 100% by W4, may be due to N in the irrigation water that was not measured or otherwise to errors in determining the yields or N concentrations. Nevertheless, increased irrigation, and presumed increased ET, greatly increased the N use by the crop and resulted in improved growth and yields. Interestingly, the water and fertility levels did not affect the relationship between grain yield and aboveground dry matter which was highly linear with a coefficient of determination of 0.982. Although this study did not report the crop water use, it demonstrates the following concepts discussed by van Keulen (1986a): (i) grain yield is approximately proportional to N uptake at lower levels of N uptake, which leads to consistent minimum N concentrations in the plant material; (ii) at higher N uptake levels the yield response is nonlinear, reflecting increasing N concentrations in the economic yield products, resulting in lower N use efficiency but probably greater protein content in the case of grains; (iii) at some point on the N application curve, yield response plateaus, indicating the limitation of some other parameter (water, light, temperature, salinity, other nutrients, etc.); and (iv) the yield-N uptake curve will extend to the point where the plant has reached the point of maximum N concentration in its tissues throughout its life cycle. The intercept of the N uptake-

![Graph](image)

Fig. 14-9. The relation between (A) corn grain yield and water and N applications; (B) grain yield relation to nitrogen uptake; (C) N uptake and N application; and (D) the efficiency of nitrogen recovery as affected by the irrigation applications (W1 dry to W4 wet) for sprinkler-irrigated corn at Kaysville, UT (Stapleton et al., 1983). The graphs were developed using the procedure outlined by van Keulen (1986a).
Table 14-3. Minimum concentrations (g kg$^{-1}$) for the major plant nutrients in the economic and the crop residue portions of crop yield for several types of crops (van Keulen, 1986b).

<table>
<thead>
<tr>
<th>Crop type</th>
<th>Economic yield component</th>
<th>Residue yield component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>P</td>
</tr>
<tr>
<td>Grains</td>
<td>10</td>
<td>1.1</td>
</tr>
<tr>
<td>Oil seeds</td>
<td>15.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Root crops</td>
<td>8</td>
<td>1.3</td>
</tr>
<tr>
<td>Tuber crops</td>
<td>4.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

N application curve (Part C of Fig. 14–9) indicates the mineralized N available from the soil which is not greatly affected by the different irrigation treatments but would vary from year to year depending on water and temperature levels, crop rotations, and other management factors. The slope of N uptake-N application represents the fertilizer recovery efficiency which depends on the type of fertilizer material, application methods, application timing, environmental factors, etc., and would normally be <80% (van Keulen, 1986a). These procedures can be extended to other nutrients as well as nutrient interactions. Van Keulen (1986b) summarized the minimum concentrations of major plant nutrients for several crops, as given in Table 14–3. The data from Table 14–3—when combined with estimates of the soil available nutrients contributing to crop uptake, and fertilizer recovery efficiency and potential dry matter yield (or estimated economic yield along with an estimated value for the harvest index)—permit the minimum fertilizer applications to be estimated (Stanford & Legg, 1984) that are necessary to produce the estimated yield level. Much information, including the previous cropping history and organic amendments, is required to precisely estimate the fertilizer requirements of a crop.

C. Salinity Effects

Salinity (soluble salts or specific ions) present in the soil or in the irrigation water solution can significantly affect crop yield as well as the relationship between crop yield and evapotranspiration. Basically, the plant transpires pure water and only pure water evaporates from the soil, leaving the soluble salts within the soil solution. These processes change both the osmotic and matric potentials within the soil profile. Childs and Hanks (1975) demonstrated that these two components of total soil water potential are additive in terms of their effects on crop transpiration and should be additive in terms of their effect on crop production. However, as discussed by Bresler and Hoffman (1986), the interpretation or prediction of the interaction of salinity and irrigation water quantity on crop production is complex, depending on the transient nature of the soil salinity patterns.

Maas and Hoffman (1977) summarized much of the existing literature on salt tolerance of crops in terms of yield response to salinity as shown in
Fig. 14–10. Salt tolerance classification of crops (Maas & Hoffman, 1977).

Fig. 14–10. Recently, Maas (1986) updated the previous summary of data on crop salt tolerance. The salt tolerance of crops is defined as

\[ Y_{Y_m^{-1}} = 1 - B_s \text{ (EC}_e - A_s) \quad \text{[for EC}_e \geq A_s] \]  
\[ Y_{Y_m^{-1}} = 1 \quad \text{[for EC}_e \leq A_s] \]

where \(A_s\) is the salinity threshold in dS m\(^{-1}\), \(B_s\) is the sensitivity of the crop to salinity above the threshold level in m dS\(^{-1}\), and \(\text{EC}_e\) is the electrical conductivity of saturation extract in dS m\(^{-1}\). The saturated extract electrical conductivity of soils is generally considered to be about one-half of the electrical conductivity of the actual soil solution for mineral soils. These relationships are applicable where Cl is the main ion affecting yield. The main difficulty in using these descriptions of salt tolerance of crops is that the electrical conductivity of the soil is dynamic depending on the salinity of the applied irrigation water and/or the received precipitation (Meiri, 1984), crop water extraction profiles (Raats, 1974; Hoffman & van Genuchten, 1983), the initial salinity profiles, and the soil chemical reactions (particularly for irrigation waters high in sulfates and carbonates) (Rhoades & Merrill, 1976).

Feinerman et al. (1984) developed procedures to compute corn yield in relation to applied water, salinity, and application uniformity using the salt tolerance concepts of Maas and Hoffman (1977) and using both steady-state soil salinity and transient cases based on the model of Bresler (1967). Lety et al. (1985) included effects of plant adjustments to the root zone salinity such that even with limited irrigation applications using saline water, leaching (drainage from the root zone) could occur if plant water use was reduced due to the salinity of the soil water solution. Lety and Dinar (1986) presented yield relationships for several crops based on their previously developed procedures. Solomon (1985) developed procedures similar to Lety et al. (1985) to
predict crop production from water and salinity relationships and included two functions for water: (i) for the increasing production side of the curve and (ii) for describing the aeration effects (and/or nutrient leaching) on crop production due to excessive irrigation.

Bresler and Hoffman (1986) demonstrated for a variety of crops that crop yield (both dry matter and economic yield) was related to applied irrigation water and the water salinity by using steady-state and transient models. The major effect of soil salinity was to reduce plant water uptake as determined by the root zone water potential (total of matric and osmotic potentials). They illustrated the difficulties of predicting the dynamic characteristics of leaching using the steady-state model. Bresler (1987) postulated that the transient model could explain both crop yield response to irrigation quantity and water quality as well as specific crop salt tolerances based on the limiting (or lowest possible) value for total plant root potential.

The effects of irrigation water salinity on crop yields are illustrated in Fig. 14–11 based on the concepts developed by Bresler (1987), Bresler and Hoffman (1986), and Letey et al. (1985). With nonsaline irrigation water, the crop yield response is similar to that previously described; as the irrigation water salinity increases to moderate levels, the yield declines almost in proportion to the salinity level (Fig. 14–11). But as the irrigation water salinity continues to increase, the generally linear lines become pronounced curves. Bresler (1987) emphasized that this relationship would not be applicable to all conditions due to aeration or leaching of plant nutrients as irrigation applications became excessive to overcome the irrigation water salinity.

Bresler and Hoffman (1986) analyzed both dry matter production (above ground) and economic yield components from a variety of experiments dealing with salinity. They reported that economic yield was highly correlated to dry matter yield \( r^2 = 0.99 \), and thus, the relationship between these yield components was not differentially affected by salinity. Hanks et al. (1978) reported that both grain yield and dry matter yield of corn were linearly

![Diagram](image)

**Fig. 14–11.** Example estimates of relative crop yield in relation to relative water application and irrigation water salinity. Adapted from Bresler (1987).
related to evapotranspiration over a wide range of water and salinity treatments, implying that grain yield was also linearly related to dry matter yields. Figure 14–12 illustrates this relationship for corn using the data from four locations for 2 yr at each location, with four to six different salinity and irrigation levels applied at each location as well as several corn varieties and several irrigation deficit period schedules (Stewart et al., 1977). The relationship between grain yield (dry) and aboveground dry matter is similar to that proposed by Slabbers et al. (1979) and accounts for more than 84% of the variance in the grain yields (note when the 1975 data shown in Fig. 14–12 for Fort Collins and Yuma were deleted due to an early freeze and extreme heat stress at the respective locations in that year, the coefficient of determination increased to 0.90). Similar results were obtained when the relationships between grain sorghum yield and dry matter yield and seed cotton yield and dry matter were examined using data from Maas et al. (1986) and Russo and Bakker (1987), respectively, from a variety of water and salinity treatments. The cotton yield (seed cotton or lint) in relationship to dry matter yield is slightly nonlinear, however, as verified with data from Davis (1983).

D. Effects of Water Deficits at Critical Crop Growth Periods

Water deficits at critical crop development stages have been reported to adversely affect crop yields (Hagan et al., 1959). The effects of water deficits and/or irrigation additions at specific crop growth stages were summarized by Salter and Goode (1967) for many types of crops (Table 14–4). In general, crop water deficits during floral initiation or anthesis have been reported to have the greatest effects on crop economic or grain yields through reductions in seed or grain numbers, while water deficits after anthesis through grain filling generally reduce seed or grain mass. Doorenbos and Kassam (1979) provided summary information regarding effects of critical periods of water deficits on crop production. One of the major problems

![Graph](https://example.com/graph.png)

Fig. 14–12. Relationship between corn grain yield (dry) and aboveground dry matter. Data from Stewart et al. (1977).
Table 14-4. Summary of the most critical growth periods for water deficits of selected crops on their production (Salter & Goode, 1967).

<table>
<thead>
<tr>
<th>Crop</th>
<th>Most critical periods for water deficits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>Shooting (elongation of internodes)</td>
</tr>
<tr>
<td></td>
<td>Earing (emergence of the ear from the boot)</td>
</tr>
<tr>
<td>Corn</td>
<td>Flowering</td>
</tr>
<tr>
<td></td>
<td>Early grain formation</td>
</tr>
<tr>
<td>Barley</td>
<td>Shooting</td>
</tr>
<tr>
<td></td>
<td>Earing</td>
</tr>
<tr>
<td>Oat</td>
<td>Heading</td>
</tr>
<tr>
<td></td>
<td>Flowering</td>
</tr>
<tr>
<td>Rye</td>
<td>Flowering</td>
</tr>
<tr>
<td></td>
<td>Early grain formation</td>
</tr>
<tr>
<td>Sorghum</td>
<td>Booting (end of shooting stage just prior to the emergence of the head)</td>
</tr>
<tr>
<td></td>
<td>Heading</td>
</tr>
<tr>
<td>Rice</td>
<td>Flowering</td>
</tr>
<tr>
<td>Cereal summary</td>
<td>Main effects of water deficits at critical periods appear to mainly affect the number of grains with some effects on other yield components like tiller number and grain mass.</td>
</tr>
<tr>
<td>Peas</td>
<td>Flowering</td>
</tr>
<tr>
<td></td>
<td>Pod filling</td>
</tr>
<tr>
<td>Soybean and other beans</td>
<td>Flowering</td>
</tr>
<tr>
<td></td>
<td>Pod set</td>
</tr>
<tr>
<td>Peanuts</td>
<td>Flowering</td>
</tr>
<tr>
<td></td>
<td>Seed development</td>
</tr>
<tr>
<td>Annual legume summary</td>
<td>Differing results were reported but generally indicated that flowering and pod development were the most critical periods.</td>
</tr>
<tr>
<td>Tomato, pepper, and cucumber</td>
<td>Start of fruit set onwards</td>
</tr>
<tr>
<td>Annual fruit crop summary</td>
<td>Basically, all annual fruit crops are sensitive to water deficits at the time that the first fruits start to develop</td>
</tr>
<tr>
<td>Cotton</td>
<td>Flowering</td>
</tr>
<tr>
<td></td>
<td>Boll development</td>
</tr>
<tr>
<td>Flax</td>
<td>Vegetative growth (fiber production)</td>
</tr>
<tr>
<td></td>
<td>Flowering (seed production)</td>
</tr>
<tr>
<td>Safflower</td>
<td>Rosetting</td>
</tr>
<tr>
<td></td>
<td>Flowering</td>
</tr>
<tr>
<td></td>
<td>Seed filling</td>
</tr>
<tr>
<td>Sunflower</td>
<td>Heading</td>
</tr>
<tr>
<td></td>
<td>Grain filling</td>
</tr>
<tr>
<td>Fiber and seed crop summary</td>
<td>Generally, like the cereals, these crops show critical periods of development near to or at flowering.</td>
</tr>
<tr>
<td>Sugar beet</td>
<td>No critical stages (root production)</td>
</tr>
<tr>
<td></td>
<td>Flowering (seed production)</td>
</tr>
<tr>
<td>Carrot</td>
<td>No critical stages</td>
</tr>
<tr>
<td>Turnip</td>
<td>Seedling (leaf and root production)</td>
</tr>
<tr>
<td></td>
<td>Prior to harvest (root production)</td>
</tr>
<tr>
<td>Cabbage</td>
<td>Head formation</td>
</tr>
<tr>
<td>Cauliflower and broccoli</td>
<td>All stages (curd production)</td>
</tr>
</tbody>
</table>

(continued on next page)
Table 14-4. Continued.

<table>
<thead>
<tr>
<th>Crop</th>
<th>Most critical periods for water deficits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biennial crop</td>
<td>Basically, the biennial crops have critical periods near to the time that the storage organ begins to develop.</td>
</tr>
<tr>
<td>summary</td>
<td></td>
</tr>
<tr>
<td>Potato</td>
<td>Tuber initiation through maturity</td>
</tr>
<tr>
<td>Onions</td>
<td>Flowering (seed production)</td>
</tr>
<tr>
<td>Tuber and bulb</td>
<td>Generally, water deficits before tuber initiation reduce the number of tubers, and water deficits during tuber formation reduce tuber size.</td>
</tr>
<tr>
<td>crop summary</td>
<td></td>
</tr>
</tbody>
</table>

involved in identifying critical periods for crops in relation to water deficits is quantifying the degree of imposed crop water deficit (or crop water stress). Many indices of crop water deficit have been used to quantify the deficit (Hiler & Clark, 1971). Several example indices include the following:

\[
(\text{ET} \frac{\text{ET}_{m}}{\text{ET}})_{i} \quad \text{(Jensen, 1968)} \quad [40] \\
[1 - (\text{ET} \frac{\text{ET}_{m}}{\text{ET}})]_{i} \quad \text{(Hiler & Clark, 1971)} \quad [41] \\
\{1 - [1 - (\text{ET} \frac{\text{ET}_{m}}{\text{ET}})]^{2}\}_{i} \quad \text{(Minhas et al., 1974)} \quad [42]
\]

where (\text{ET})_{i} is crop water use during specific crop growth period \(i\), and (\text{ET}_{m})_{i} is the crop water use during period \(i\) without any imposed crop water deficits. These indices are used in various forms of production functions (see Eq. [30], [31], [32a], etc.) in which either additive or multiplicative functions are developed (note that generally these functions are applied to grain or economic yields and not to dry matter yields). Singh et al. (1987) reported that multiple-period models did not predict wheat yields any more consistently or accurately than simpler yield-ET models. Wenda and Hanks (1981) reported similar results for corn.

The ET deficit [(\text{ET}_{m} - \text{ET}) \text{ET}_{m}^{-1}] experienced in a specific crop growth stage will seldom exceed 0.5 unless the soil water deficit is large when the growth stage is initiated. Also, the field measurement of the ET deficit is subject to large potential errors in many cases since it may be small and of short duration. Little experimental evidence has been reported that illustrates differential effects of water deficits at specific growth stages on the transpiration ratio as normalized by \(E_{o}\) or \((e^{*} - e)\). Asrar et al. (1984) did report that ET \(P^{-1}\) for wheat declined following anthesis not in proportion to decreases in \((e^{*} - e)\), but ET \(P^{-1}\) was not affected by planting density for two genotypes. If crop water deficits at specific growth stages differentially affect economic crop yields, the water deficits would have to also affect the relationship between economic yield and dry matter yield (harvest index). This topic has not been widely studied. Figure 14-12 illustrates a summary of several corn irrigation experiments where ET deficits were intentionally created in specific crop growth stages, yet no major effects of the water deficits on the partitioning of grain from the above-ground dry matter production were evident (See also Hanks et al., 1978.), although considerable vari-
ation is present in the data illustrated. However, it is apparent that certain environmental parameters can produce significantly different results, like the 1975 data from Fort Collins and Yuma as reported by Stewart et al. (1977), and that certain cultivars may be different (DeLoughery & Crookston, 1979). Consistent relationships between economic yield and dry matter yield have been determined based on data reported for grain sorghum using specific periods of osmotic stress by Maas et al. (1986) ($r^2 = 0.914$ for two varieties of grain sorghum and three growth stages), for navy beans (*Phaseolus vulgaris* L.) by Gunton and Evenson (1980) ($r^2 = 0.977$ for two growth stages and their combination), for wheat by Singh et al. (1987) ($r^2 = 0.950$ for two locations, Germany and India, with two different varieties and seven stress periods), for dwarf wheat in India by Singh and Malik (1983) ($r^2 = 0.848$ for three different stress periods), for spring wheat in Denmark by Mogensen et al. (1983) ($r^2 = 0.937$ with seven different stress periods), for cowpea [*Vigna unguiculata* (L.) Walp] by Ziska and Hall (1983) ($r^2 = 0.904$ for six irrigation levels with two fertility levels), and lima bean (*Phaseolus lunatus* L.) by Ziska et al. (1985) ($r^2 = 0.766$ for three levels of soil water depletion and two different stress periods).

E. Effects of Soil Spatial Variability and Irrigation Uniformity

With the current level of soil and irrigation science, the effects of soil spatial properties and irrigation application variability cannot be easily evaluated independently; however, their individual and/or combined effects can be estimated for specific situations. As the foregoing discussions have indicated, crop production is greatly influenced by soil water availability, with irrigation having the major effect on soil water levels. Soil properties greatly affect the processes of infiltration, root development, plant water extraction, chemical reactions, redistribution of profile soil water, and water holding capacity. Each of these processes, in turn, can affect crop production through the availability of soil water to the crop. Warrick and Nielsen (1980), Russo and Bresler (1981), and Trickler (1981) have reported that the distribution of water infiltration rates and/or hydraulic conductivity is highly skewed and likely log-normally distributed. The distribution of water held within the soil profile will likely be less skewed and more normally distributed (Cassel & Bauer, 1975; Russo & Bresler, 1981). Peck (1983) discussed soil spatial variability and its effects on water and solute transport within fields. For irrigation systems that depend on the soil for distribution (surface methods, furrow, border, flood, etc.), the distribution of soil hydraulic properties will directly affect the distribution of infiltrated water. However, the soil will redistribute the infiltrated water such that the resulting soil water distribution may be more uniform than the infiltration distribution for nonuniform irrigation applications (Hart, 1972). If the application distribution is perfectly uniform, however, then the resulting soil water storage could be less uniform due to soil variability. For pressurized irrigation distribution systems (e.g., sprinkler, drip, trickle), the distribution of the soil hydraulic properties will not greatly affect the infiltration distribution, which will depend mainly on
the water distribution from the irrigation system. Both irrigation application uniformity and soil water storage uniformity will affect the mean production from a field. The previous sections have assumed that all applied water is evenly distributed and that the water in the soil is uniformly available to the crop.

Solomon (1984) reviewed the irrigation uniformity parameters that have been used by engineers to describe the application distribution from irrigation systems such as the following:

\[
UC = 1 - (\delta \bar{Q}^{-1}) \quad \text{(Christiansen, 1942)} \quad [43]
\]

\[
DU = \frac{Q_{\text{iq}}}{\bar{Q}} \bar{Q}^{-1} \quad \text{(Cridle et al., 1956)} \quad [44]
\]

where UC is called a uniformity coefficient, DU was originally called a pattern efficiency and later called the distribution uniformity, \( \delta \) is the mean deviation of the application (\(|Q_i - \bar{Q}| \), where \( Q_i \) is an individual observation in mm), \( \bar{Q} \) is the mean application in mm, and \( Q_{\text{iq}} \) is mean of the lowest one-quarter of the applications in mm. Warrick (1983) examined these two functions and reported their characterizations in terms of the coefficient of variation (CV, \( \sigma \bar{Q}^{-1} \), where \( \sigma \) is the standard deviation of the applications in mm) and several types of application population distributions (normal, log normal, specialized power, beta, and gamma). For most of the distributions (particularly for CV \( \leq 0.5 \)), Warrick (1983) proposed that the uniformity coefficients could be estimated as follows:

\[
UC = 1 - 0.8 \text{ CV} \quad [45]
\]

\[
DU = 1 - 1.3 \text{ CV}. \quad [46]
\]

Since the spatial variability of resulting irrigation intake and the resulting spatial variability of crop water use is complex, most early research on the effects of irrigation uniformity simply considered yield effects related to irrigation uniformity. Zaslavsky and Buras (1967) used a Taylor series expansion of the yield-water relationship to determine the yield as:

\[
\bar{Y} = \bar{Y}(\bar{Q}) + 0.5 \left( \frac{\partial^2 Y}{\partial Q^2} \right) \bar{Q}^{-1} \sigma^2 \quad [47]
\]

where \( \bar{Y} \) is the mean yield in kg ha\(^{-1} \), \( Y(\bar{Q}) \) is the yield in kg ha\(^{-1} \) from the yield-water function at the mean water application, \( \left( \frac{\partial^2 Y}{\partial Q^2} \right) \bar{Q}^{-1} \sigma^2 \) is the second derivative in kg ha\(^{-1} \) mm\(^{-2} \) of the yield-water function evaluated at the mean water application, and \( \sigma^2 \) is the variance in mm\(^2 \) of the irrigation application. Equation [47] assumes that the higher order terms of the Taylor series are negligible, and it requires a twice differentiable yield-applied water function. Varlev (1976) investigated the interactions of irrigation uniformity and irrigation quantity using similar concepts and focused on "infiltrated water" contrasted to "applied water" (this implicitly brings into the function the soil spatial variability), and discussed the trade-offs be-
between irrigation uniformity and water applications to achieve optimum yield levels. Segner (1978) estimated that the marginal economic value of an increment of irrigation uniformity (UC) would be approximately one-half of the maximum income per unit land area ($Y_m$ times the commodity price). Segner (1983) extended his previous analysis to include the economic evaluation of irrigation uniformity in terms of land and water constraints. Warrick and Gardner (1983) examined the problem of soil spatial and irrigation application variability on yield and irrigation water use efficiency (yield per unit applied water). Unlike Varlev (1976) and Segner (1978), they attempted to combine the irrigation application variability and soil spatial variability distributions using the joint probability distributions which can be determined by convolution or Monte Carlo methods. Figure 14–13 shows an example from Warrick and Gardner (1983) using a log-normal irrigation distribution and a uniform distribution of available soil water for CVs of 0, 0.5, 1, and 1.5. This example indicates that any variation in irrigation application (and/or equivalent soil water infiltration) will tend to skew the yield-water function. The effect of soil infiltration and irrigation application variability is similar to the effects of increasing irrigation water salinity on crop yields (Fig. 14–11). This skewing of the linear crop yield-ET lines obtained from small plots (where properties are uniform) into curves for fields (where natural variation might be large) may be one factor to help explain the curved nature of other yield-ET and yield-applied water functions widely found in the literature. Slight irrigation or soil infiltration variability can produce nonlinear yield-ET functions even though the basic yield function is exactly linear. Additional discussions of the effects of irrigation or soil infiltration variability on crop yield are found in Stern and Bresler (1983), Solomon (1984), Feinerman et al. (1984), Lety et al. (1984), and Lety (1985).

![Hypothetical Example Adapted from Warrick and Gardner (1983)](image)

Fig. 14–13. Hypothetical relationship between crop yield and applied irrigation water for several application variabilities (expressed in terms of the CV, coefficient of variation) for log-normally distributed irrigations and uniform soil water variability (Warrick & Gardner, 1983).
III. CROP PRODUCTION-IRRIGATION RELATIONSHIPS

The relationships between net assimilation and transpiration, dry matter production and transpiration, and economic crop production and crop water use have been discussed along with descriptions of the effects of evaporative demand, fertility, salinity, and soil and irrigation variability. The basic relationship between net assimilation and transpiration depends on the photosynthetic pathway, plant diffusion resistances, and specific environmental factors. Dry matter production is controlled by the same factors that affect net assimilation, although the interpretation of field data is difficult because of the inability to measure field transpiration as well as the usual omission of the measurement of root dry matter. Economic production can be estimated from the dry matter production in most instances. The relationships between economic yield and evapotranspiration are basically empirical and depend largely on the above described factors as well as the evaporative demand. Severe water deficits in certain critical crop development periods can interfere with the crop development, in particular reproductive processes, and reduce economic crop yield, but the effects may be difficult to precisely define due to the interactions of water deficits between several crop growth stages. Nutrition, if inadequate, can limit production as well as reduce the efficiency of crop water use. Salinity, soil spatial variability, and irrigation application variability all act similarly to skew the relationship between crop production and water application and will generally reduce the mean crop production on a field basis.

The interpretation or the prediction of the effects of irrigation on crop production is complex. Obviously, the effects of irrigation on crop production must be accurately predicted to permit economic analyses of irrigation systems, irrigation management, and water resource allocation decisions. The relation of irrigation to crop production is essentially site specific. Yaron and Bresler (1983) and Vaux and Pruitt (1983) provide excellent interpretations and reviews along with discussions of the limitations of productions functions for economic evaluation of irrigation water applications.

The graphical presentation of the relationship between crop production and the field water supply presented by Stewart and Hagan (1973) illustrates the concepts discussed in this chapter (Fig. 14-14). They defined the field water supply (FWS) to be the sum of the soil water in the profile at planting that will become available to the crop during the season (ASW), the gross seasonal irrigations ($Q$) (also would include preplant irrigations if not included in ASW), and the rainfall ($R$) received during the season. This example shows a case where the sum of ASW and $R$ is 250 mm and where 1150 mm of irrigation water is needed to be applied (with the implied application efficiency, application uniformity, inherent soil variability, and irrigation water salinity) to obtain maximum crop production, $P_m$ and $Y_m$. The example $P_m$ and $Y_m$ are 24 and 11 Mg ha$^{-1}$ for aboveground dry matter and grain yield (dry basis), respectively. Note that the points ($P$, $Q_o$) and ($Y$, $Q_o$) represent the yields without irrigation (dryland). The slope of the dry matter production line ($S_p$) is determined by the species of the crop (basically, the $k_d$ value) and
Fig. 14–14. Hypothetical example illustrating a relationship between dry matter and grain production and field water supply.

the environment (evaporative demand as characterized by either potential ET or vapor pressure deficit). The slope of the grain yield line ($S_y$) is determined by the partitioning between dry matter and economic yield and $S_p$. The deviation of the dashed curves from the lines represents the combined effects of the irrigation hydrology (runoff, deep percolation, soil water recharge, spray evaporation, drift, etc.) with the effects of the irrigation water salinity, irrigation application uniformity, and the spatial variability of the soil physical parameters. Vaux and Pruitt (1983) discussed these concepts in detail and reported that relationships like those illustrated in Fig. 14–14 closely resembled the results reported by a number of investigators. Martin et al. (1984) developed procedures to estimate the contribution of irrigation to evapotranspiration, which was then related with production functions to grain yield. The relationship between grain yield and field water supply shown for the specific example in Fig. 14–14 (which is site, crop, and irrigation specific) demonstrates the following conditions: (i) maximum water use efficiency ($Y ET^{-1}$) occurs at the point $(Y_m, ET_m)$; (ii) maximum irrigation water use efficiency ($Y Q^{-1}$) occurs at a value of $Q$ of about 600 mm for this example, which is considerably less than the 1150 mm necessary to produce maximum grain yield (this value can be graphically determined by the tangent on the curve to a line constructed through the origin); and (iii) assuming a constant water cost, the maximum net profit will normally occur at a value of FWS exceeding $ET_m$ but $<Q_m$ (unless water is free) and will decrease as the water price increases for fixed land but increase with higher fixed production costs (Yaron & Bresler, 1983). Generally, the net profit will be rather insensitive to a relatively broad range in applied water (likely to be ±25–50 mm in this example) and, therefore, the grower would likely choose the higher irrigation applications (if sufficient water is available) within this range to
avoid risk associated with critical water deficits as well as the other intangible factors.

Future research into crop production as affected by irrigation will not benefit from developing empirical production functions, except where the basic knowledge of the irrigation uniformity or soil variability or crop-soil rooting interactions is deficient. Procedures are available to estimate \( P_m \) and \( \text{ET}_m \) as well as the relationships between \( P \) and \( T \) and \( Y \) and \( P \) (e.g., Doorenbos & Kassam, 1979; Doorenbos & Pruitt, 1977; Tanner & Sinclair, 1983; Feddes, 1986; van Kuenlen & Wolf, 1986). Additional improvements in the quantification of the effects of water deficits at critical crop growth stages are needed. As the development of comprehensive crop growth models increases, the irrigation economic analyses and real-time irrigation decisions (Swaney et al., 1983) can be accomplished with expert systems (Lemon, 1986) that rely on crop simulation. Few current crop simulation models contain the sophistication to deal with all of the simultaneous problems related to water, salinity, fertility, insects, diseases, soil chemical and physical limitations, and irrigation dynamics—as well as the environmental variability—but the future for their application to irrigation management problems appears promising.

**APPENDIX**

**Symbols**

\[
A = \text{Assimilation flux density, kg (CH}_2\text{O) m}^{-2} \text{d}^{-1}, \text{mmol (C}_2\text{O) m}^{-2} \text{d}^{-1}, \text{or kg (CO}_2\text{) m}^{-2} \text{s}^{-1}
\]

\[
A_s = \text{Salinity-yield threshold, dS m}^{-1}
\]

\[
B = \text{Crop-specific coefficient}
\]

\[
B_s = \text{Sensitivity factor for salinity, m dS}^{-1}
\]

\[
B' = \text{Correction factor for leaf shading}
\]

\[
C = \text{Crop vector}
\]

\[
C_a = \text{Atmospheric CO}_2\text{ concentration, kg m}^{-3}
\]

\[
C_s = \text{CO}_2\text{ concentration inside substomatal cavity, kg m}^{-3}
\]

\[
CV = \text{Coefficient of variation}
\]

\[
DU = \text{Distribution uniformity}
\]

\[
E = \text{Soil water evaporation, mm or mm d}^{-1}
\]

\[
E_c = \text{Seasonal mean pan evaporation within screened enclosure, mm d}^{-1}
\]

\[
E_o = \text{Potential ET, mm d}^{-1}
\]

\[
EC_e = \text{Electrical conductivity of saturated extract, dS m}^{-1}
\]

\[
ET = \text{Evapotranspiration, mm or mm d}^{-1}
\]

\[
ET_m = \text{Maximum ET without water deficits, mm}
\]

\[
H = \text{Seasonal mean daily relative humidity, 00}
\]

\[
H_i = \text{Harvest index}
\]

\[
H_{ia} = \text{Adjusted harvest index}
\]

\[
\text{LAI}_D = \text{Sunlit leaf area index}
\]

\[
L_T = \text{Effective transpiration leaf area index}
\]
\[ M = \text{Miscellaneous vector} \]
\[ P = \text{Crop dry matter production, kg per container, kg ha}^{-1} \text{ or Mg ha}^{-1} \]
\[ P_a = \text{Aboveground dry matter yield, Mg ha}^{-1} \]
\[ P_b = \text{Barometric pressure, kPa} \]
\[ P_m = \text{Maximum dry matter production without water deficits, kg ha}^{-1} \text{ or Mg ha}^{-1} \]
\[ P_o = \text{Dry matter production required to initiate economic production, Mg ha}^{-1} \]
\[ P_r = \text{Root dry matter yield, Mg ha}^{-1} \]
\[ P_s = \text{Stover dry matter yield, Mg ha}^{-1} \]
\[ P_t = \text{Total dry matter yield, Mg ha}^{-1} \]
\[ Q = \text{Irrigation application, mm} \]
\[ \bar{Q} = \text{Mean irrigation application, mm} \]
\[ Q_m = \text{Irrigation application required to produce the maximum yield, mm} \]
\[ R = \text{Incident solar radiation, W m}^{-2} \]
\[ S_b = \text{Dry matter yield-evapotranspiration slope, Mg ha}^{-1} \text{ mm}^{-1} \]
\[ S_e = \text{Economic yield-evapotranspiration slope, Mg ha}^{-1} \text{ mm}^{-1} \]
\[ T = \text{Transpiration, kg m}^{-2} \text{ d}^{-1}, \text{ mmol m}^{-2} \text{ s}^{-1}, \text{ kg per container, or mm d}^{-1} \]
\[ T_m = \text{Seasonal transpiration without water deficits, mm} \]
\[ T_T = \text{Seasonal transpiration, mm} \]
\[ UC = \text{Christiansen's uniformity coefficient} \]
\[ W = \text{Weather vector} \]
\[ W_a = \text{Atmospheric water vapor concentrations, kg m}^{-3} \]
\[ W_s = \text{Atmospheric water vapor concentrations inside substomatal cavity, kg m}^{-3} \]
\[ Y = \text{Economic yield, kg ha}^{-1} \text{ or Mg ha}^{-1} \]
\[ Y_m = \text{Maximum economic yield without water deficits, kg ha}^{-1} \]
\[ a = \text{Molecular mass ratio of CH}_2\text{O to CO}_2, 0.68 \]
\[ b = \text{Conversion factor for CH}_2\text{O to biomass, 0.33 to 0.83} \]
\[ c = \text{CO}_2 \text{ gradient factor } \left( \rho - \rho_s \rho^{-1} \right) \]
\[ e = \text{Vapor pressure, kPa} \]
\[ e^* = \text{Saturated vapor pressure at the crop temperature, kPa} \]
\[ e^{*} = \text{Saturated vapor pressure at the air temperature, kPa} \]
\[ k, k_a, k_d = \text{Crop-specific coefficients, kPa} \]
\[ k_a = \text{Crop-specific coefficient, \% (RH)} \]
\[ m = \text{Crop-specific coefficient, mm d}^{-1} \text{ or kg ha}^{-1} \text{ d}^{-1} \]
\[ m_e = \text{Crop-specific coefficient within screened enclosure, mm d}^{-1} \]
\[ n = \text{Crop-specific coefficient, kg kg}^{-1} \]
\[ r = \text{Leaf diffusion resistance to H}_2\text{O, s m}^{-1} \]
\[ r' = \text{Leaf diffusion resistance to CO}_2, \text{ s m}^{-1} \]
\[ \delta = \text{Mean irrigation application deviation, mm} \]
\[ \rho = \text{Ambient CO}_2 \text{ concentration, mg kg}^{-1} \]
\[ \sigma = \text{Standard deviation of irrigation application, mm} \]
\[ \rho_s = \text{CO}_2 \text{ concentration inside substomatal cavity, mg kg}^{-1} \]
\[ \rho_a = \text{Density of air, kg m}^{-3} \]
\[ \epsilon = \text{Molecular mass ratio of water vapor to air, 0.622} \]
\[ \lambda = \text{Constant (Lagrange multiplier)} \]
\[ l_i = \text{Crop-specific coefficient for growth stage } i \]
\[ \theta = \text{Soil vector} \]
REFERENCES


