Linear combinations of n spectral bands form physically significant indices in n-dimensional space. The 2-dimensional (2-D) perpendicular vegetation index (PVI) of Richardson and Wiegand and the 4-D tasseled cap of Kauth and Thomas are special cases of n-space indices. A procedure for calculating the coefficients of n-space indices is described. Spectra from 12 wheat and two bare soil (wet and dry) plots were multiplied point by point (at 1-nm intervals) by response functions representing five satellite sensors. Reflectance values were obtained for each band for each sensor (atmospheric effects and sensor characteristics such as noise, resolution, and calibration, were not considered). N-Space indices were calculated for various band combinations for the several sensors and their dynamic range for a 0–100% change in vegetation was compared. A 6-D vegetation index (greenness) calculated using six of the thematic mapper bands had the greatest dynamic range, followed closely by two 5-D and one 4-D greenness from the same sensor. The 2-D greenness using bands 4 (near-IR) and 7 (mid-IR) of the thematic mapper had a greater dynamic range than any band combination of the other four satellite sensors. The 4-D greenness of the Landsat-4 MSS and the 3-D index of the SPOT HRV were similar. The 2-D indices from the AVHRR sensors on NOAA-6 and NOAA-7 changed less with vegetation changes than did the other three.

Introduction

Kauth and Thomas (1976) proposed a transformation that used the four Landsat MSS bands in linear combinations to produce four indices called brightness (BR), greenness (GN), yellowness (YE), and nonsuch (NS). Concurrently, Richardson and Wiegand (1977) used two Landsat bands (IR and red) to develop a 2-dimensional perpendicular vegetation index (PVI). Later Wiegand and Richardson (1982) introduced a soil line index (SLI) as a companion to the PVI, and compared their 2- with the 4-dimensional indices of Kauth and Thomas (1976). These indices are special cases of a class of spectral indices, linear combinations of n spectral bands, in n-dimensional space. This class of indices is especially useful for the discrimination of vegetation from soil background, the primary purpose for the development of the 2- and 4-space cases. The derivation of the 2-D PVI essentially consists of finding the perpendicular (orthogonal) distance from a line (representing soils) to a point (representing vegetation). As n increases, the complexity of the computation increases. Kauth and Thomas (1976) used the Gram–Schmidt process (Freiberger, 1960) to obtain four orthogonal linear combinations of the four Landsat MSS bands. From a Landsat image, they selected clusters of pixels identified as soil, and other clusters identified as vegetation, and calculated the appropriate unit vectors which form the coefficients of the indices. Subsequently, statistical procedures such as principal component analysis have been used to develop the 4-D indices (they can readily produce indices in n dimensions). Statistical procedures are well suited for analyzing large area scenes (such as a Landsat image) where little a priori knowledge of the soils and vegetation is
available, and only large scale information is desired. On the other hand, if smaller areas such as individual fields are of interest, and reflectance information on local soils is available, the Gram–Schmidt process is a useful procedure for obtaining the unit vectors.

Agricultural applications of remote sensing require detailed information concerning the relationship between agronomic parameters and remotely sensed data. Instruments such as the four-band Exotech Model 100A\textsuperscript{1,2} that mimics the four Landsat MSS bands, the three-band Ideas Inc. Mark II\textsuperscript{1} that mimics the thematic mapper bands 3, 4, and 5 (Tucker et al., 1981), and the eight-band Barnes 12-1000 MMR\textsuperscript{1} that mimics the seven Thematic Mapper bands plus an additional band at 1.15–1.3 \(\mu\)m (Robinson et al., 1981) are currently in use at a number of field sites. The analysis of data from these instruments can be facilitated by use of indices derived from linear combinations of bands.

The purpose of this report is to describe how the coefficients for \(n\)-space indices can be calculated using the Gram–Schmidt process with a minimum of data points. Mathematicians may find the development elementary. However, others may find a detailed description useful, since remote sensing research involves a variety of disciplines. An example of the usefulness of these indices is given by calculating \(n\)-dimensional greenness (a measure of the amount of green vegetation) for various combinations of bands representative of the Landsat-4 MSS, Thematic Mapper, SPOT HRV, and the AVHRR of NOAA-6 and NOAA-7 sensors. The dynamic range of greenness values was used to rank the various sensors, and band combinations within sensors, as to their ability to discriminate vegetation from the soil background.

The Concept of \(n\)-Space Indices

The concept of \(n\)-space is difficult to visualize if \(n\) is greater than three. However a 2-dimensional index such as the PVI is relatively easy to visualize and will be used to demonstrate the physical basis for \(n\)-space indices.

Richardson and Wiegand (1977) showed that a plot of MSS7 and MSS5 (a near-IR and a red band of the Landsat multispectral scanner) for soils would fall on a straight line. As vegetation grows on the soil, the red radiance decreases and the near-IR radiance increases. A vegetation point would lie away from the soil line with the perpendicular distance from the point to the soil line being a measure of the amount of vegetation present. Figure 1 shows a soil line with two soil and three vegetation points. Points A and B represent dry and wet soil respectively. Points C and D represent points with vegetation covering only part of the soil. The same amount of vegetation is present, but the soil surface is dry (highly reflecting) for point C and wet (less reflective) for point D. Since both points have the same perpendicular distance from the soil line, they have the same PVI value. Point E represents full vegetative cover and is essentially unaffected by soil background changes. (Jackson et al., 1983a, discussed how soil background may affect 2- and 4-dimensional indices because of differences in transmission of IR
and red radiation through vegetation). In
Fig. 1, the distances from point F to
the intersection of the lines from points
C, D, and E and the soil line is called the
soil line index (SLI) by Wiegand and
Richardson (1982).

The 4-space tasseled cap transform-
ation of Kauth and Thomas (1976) is based
on the same principles as the 2-dimen-
sional indices of Richardson and Wiegand.
A soil line is as fundamental in 4-D as in
2-D. Greenness is the orthogonal distance
from the soil line to a vegetation point.
The third dimension (yellowness) is or-
thogonal to both greenness and bright-
ness, and the fourth dimension (called
nonsuch because no features were evi-
dent) is orthogonal to the first three. The
coefficients of brightness, greenness, etc.,
are unit vectors that indicate direction.

Kauth and Thomas (1976) showed that
brightness and greenness contained al-
most all of the variation within a sample
segment, and suggested that shifts in yel-
lowness and nonsuch were diagnostic of a
physical state of the atmosphere. The
average yellowness over a segment forms
the basis of the XSTAR haze correction
algorithm of Lambeck et al. (1978). Kauth
et al. (1979) stated that nonsuch primarily
contains noise. Jackson et al. (1983a)
showed that yellowness and nonsuch were
essentially independent of vegetation
changes and that yellowness was sensitive
to haze conditions and nonsuch was sensi-
tive to water vapor absorption. Jackson
et al. (1983b) suggested that yellowness
and nonsuch could be used to correct for
atmospheric path radiance and vapor ab-
sorption. Brightness and greenness have
proved useful for evaluating soil and
vegetation features in Landsat data (Kauth
et al., 1979; Thompson and Wehmanen,
1980).

The 2-space PVI and SLI of Richard-
son and Wiegand (1977) and Wiegand
and Richardson (1982) can be considered
as 2-space greenness and brightness. Since
these terms are rather descriptive, it is
suggested that they be used with a pre-
fase, indicating the number of bands used
in their computation. Further work with
yellowness and nonsuch may clarify their
use as indicators of haze and water vapor
in the atmosphere. In time, more ap-
propriate names may be in order. The
mid-IR bands of the Thematic Mapper,
which are anticipated to be sensitive to
liquid water within plant leaves, may
prove to be “wetness” indices.3

Calculation of n-Space Coefficients

The number of dimensions (n) avail-
able in spectral space is the number of
wavelength intervals (or bands) for which
data are available. The number of spectral
indices (m) that may be calculated is also

3C. L. Wiegand, personal communication.
equal to the number of bands \((n)\). However, there is no requirement that \(n\) indices be calculated, only that the number \(m\) cannot exceed \(n\) if all indices are to be orthogonal. Often, just the first two indices are of interest. In this development, \(m + 1\) data points are required to specify \(m\) indices: however, if \(m = n\), the \((m + 1)\)th point can be arbitrary.

The \(n\)-space coefficients are unit vectors that give direction, and thus vector notation is appropriate. However, the dot product is the only vector manipulation necessary, and, since forming a dot product results in a scalar, the development can be written largely in algebraic terms. Thus, vectors will be discussed where necessary, but the algebraic forms of equations will be stressed to facilitate both comprehension and computation. The terms brightness, greenness, and yellowness, as used by Kauth and Thomas (1976), will be used here.

To obtain the first index, an equation for a line through the soil data points must be derived. A minimum of two soil points are required, with points differing considerably in reflectance preferred (e.g., wet and dry surfaces). The differences between the dry \((X_{sd})\) and the wet \((X_{sw})\) soil points are

\[
b_i = (X_{sd} - X_{sw})_i
\]

for each of \(n\) bands. The vector \((b_1, b_2, \ldots, b_n)\) is normalized to form a unit vector by dividing each of its components by the normalization factor \(B\), where

\[
B = \left( \sum_{i=1}^{n} b_i^2 \right)^{1/2}.
\]

Then,

\[
A_{1,i} = b_i / B
\]

are the coefficients of brightness. Brightness can be expressed as

\[
BR = A_{1,1}X_1 + A_{1,2}X_2 + \cdots + A_{1,n}X_n,
\]

where \(X_i\) represents values for a data point in the \(i\)th band.

Calculation of the second index (greenness) begins by choosing a data point that represents green vegetation and forming the differences between that point and any point on the soil line \((X_g - X_s)_i\).

\[
g_i = (X_g - X_s)_i - D_{2,1}A_{1,i},
\]

where

\[
D_{2,1} = \sum_{i=1}^{n} (X_g - X_s)_i A_{1,i}.
\]

This procedure, called the Gram–Schmidt process (Freiberger, 1960), insures that the vector \((g_1, g_2, \ldots, g_n)\) is orthogonal to the soil line vector \((b_1, b_2, \ldots, b_n)\). The subscripts of \(D\) indicate that it is associated with the second index (greenness) and also the first (brightness).

The normalization factor is

\[
G = \left( \sum_{i=1}^{n} g_i^2 \right)^{1/2}.
\]

The coefficients for the second index (greenness) are

\[
A_{2,i} = g_i / G,
\]
SPECTRAL INDICES IN N-SPACE

and greenness can be calculated from

\[ GN = A_{2,1}X_1 + A_{2,2}X_2 + \cdots + A_{2,n}X_n. \] (9)

The third index (yellowness) must be orthogonal to both brightness and greenness. Choose an appropriate data point and form the differences \((X_y - X_s)_i\). Then

\[ y_i = (X_y - X_s)_i - (D_{3,1}A_{1,i} + D_{3,2}A_{2,i}); \] (10)

the two \(D\) terms must be evaluated before calculating \(y_i\). The first is denoted \(D_{3,1}\) because it refers to the third (yellowness) and the first (brightness) indices, and the second as \(D_{3,2}\) because it refers to the third and the second (greenness). These terms are evaluated from the equations

\[ D_{3,1} = \sum_{i=1}^{n} (X_y - X_s)_i A_{1,i} \] (11)

and

\[ D_{3,2} = \sum_{i=1}^{n} (X_y - X_s)_i A_{2,i}. \] (12)

The normalization factor is

\[ Y = \left( \sum_{i=1}^{n} y_i^2 \right)^{1/2}. \] (13)

The coefficients are

\[ A_{3,i} = y_i/Y, \] (14)

and the equation for yellowness is

\[ YE = A_{3,1}X_1 + A_{3,2}X_2 + \cdots + A_{3,n}X_n. \] (15)

This procedure can be generalized to calculate \(m\) indices using \(n\) bands \((m \leq n)\). The dot products can be written

\[ D_{k,j} = \sum_{i=1}^{n} (X_k - X_s)_i A_{j,i} \] (16)

for \(k = 1\) to \(m\) and \(j = 1\) to \(k - 1\). If \(k = 1\) (brightness), \(j = 0\), and \(D_{k,j} = 0\), thus Eq. (1) has only the difference of the soil points on the right-hand side. It follows that the \(m\)th index (called \(t\) for convenience, with no physical significance attached to the symbol) is

\[ t_i = (X_m - X_s)_i - (D_{m,1}A_{1,i} + D_{m,2}A_{2,i} + \cdots + D_{m,j}A_{j,i}). \] (17)

The normalization is as before,

\[ T = \left( \sum_{i=1}^{n} t_i^2 \right)^{1/2}. \] (18)

The coefficients of the \(m\)th index are

\[ A_{m,i} = t_i/T \] (19)

and the \(m\)th \(n\)-space index \((I_m)\) is

\[ I_m = A_{m,1}X_1 + A_{m,2}X_2 + \cdots + A_{m,n}X_n. \] (20)

All indices should be orthogonal. This can be checked by calculating dot products of the various coefficients, i.e.,

\[ \sum_{i=1}^{n} A_{k,i}A_{j,i} = 0 \text{ for } k = j \]

\[ = 1 \text{ for } k = j. \] (21)
A numerical example is given in the Appendix.

Reflectance Data

Spectra were obtained over 12 wheat plots and two bare soil (one wet and one dry) plots with a Barnes Model 12-550\textsuperscript{1} field spectrometer operated by a team from the NASA/Goddard Space Flight Center. The wheat plots had been planted at different times to provide different growth stages. Measurements reported here are from three plots. Within each planting, subplots were delineated, and four irrigation treatments were imposed to induce different stress conditions. The 12 plots represented a wide range of green phytomass levels.

The field spectrometer was positioned about 2 m above the surface of each plot. Spectral reflectance data were obtained from 0.4 to 2.5 \( \mu \text{m} \) and were later interpolated to 1-nm increments. Data for one wheat plot are shown as the dotted line in Figure 2.

Response functions were obtained from several sources. The Landsat-4 MSS and the Thematic Mapper functions were from Markham and Barker (1982), those for SPOT from Begni (1982), and those for the NOAA satellites from Schneider et al. (1981). For reference, the wavelength intervals for the visible, near-IR, and mid-IR bands of five satellites are given in Table 1. Note that band 6 of the Thematic Mapper is in the thermal-IR, whereas band 7 is in the mid-IR. In all cases the response functions were digitized into 1-nm increments as were the spectral data. Response functions for six of the seven TM bands overlay the spectra in Fig. 2.

The reflectance for a particular band was obtained by summing the product of the spectra and a response function at each nanometer and dividing this sum by the sum of the response function values. Denoting the spectral data as \( s(\lambda) \) and the response function as \( r(\lambda) \), the reflectance in band \( i \) is

\[
\rho(\lambda)_i = \frac{\sum_{\lambda_1}^{\lambda_2} s(\lambda) r(\lambda)_i}{\sum_{\lambda_1}^{\lambda_2} r(\lambda)_i},
\]

(22)

FIGURE 2. Response functions for the visible (1,2,3), near-IR (4), and mid-IR (5,7) bands of the thematic mapper, with spectral reflectance data for wheat superimposed as a dotted line. Numbers on the ordinate also apply to the reflectance data.
TABLE 1 Wavelength Intervals (WLI) for Visible, Near-IR, and Mid-IR Bands of Five Satellite Sensors

<table>
<thead>
<tr>
<th>Band</th>
<th>WLI (μm)</th>
<th>Band</th>
<th>WLI (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>THEMATIC MAPPER</strong></td>
<td></td>
<td><strong>SPOT HRV</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.45–0.52</td>
<td>1</td>
<td>0.50–0.59</td>
</tr>
<tr>
<td>2</td>
<td>0.52–0.60</td>
<td>2</td>
<td>0.61–0.68</td>
</tr>
<tr>
<td>3</td>
<td>0.63–0.69</td>
<td>3</td>
<td>0.79–0.89</td>
</tr>
<tr>
<td>4</td>
<td>0.76–0.90</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.55–1.75</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10.4–12.5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2.08–2.35</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

| **NOAA-6 and 7 AVHRR** |          | **LANDSAT-4 MSS** |          |
| 1      | 0.58–0.68 | 1      | 0.5–0.6   |
| 2      | 0.72–1.1  | 2      | 0.6–0.7   |
| 3      | 0.7–0.8   | 3      | 0.7–0.8   |
| 4      | 0.8–1.1   | 4      | 0.8–1.1   |

*The WLI for the NOAA-6 and NOAA-7 AVHRR are essentially the same.*

where $\lambda_1$ and $\lambda_2$ are the wavelength limits of the $i$th band.

The reflectance was calculated for each band of each satellite for which response functions were available, and for each of the 12 wheat plots and the two soil plots. These data were used to calculate $n$-space coefficients of brightness and greenness.

Evaluation of the amount of green leaf area or total green phytomass can best be made by destructive sampling. Since it was not feasible to remove all the vegetation within the field of view of the spectrometer during this experiment, the determination of the exact amount of vegetation seen by the instrument was not possible. A relative amount of green vegetation was estimated by calculating the 4-space greenness factor using reflectance data for a hand-held Exotech Model 100A radiometer. The 4-space greenness factor for each plot was normalized by subtracting the factor for bare soil and dividing by the difference between the largest 4-space greenness factor of the 12 wheat plots and that for bare soil. This estimate of the relative amount of green vegetation was used to compare the various $n$-space indices calculated from the reflectance calculations.

**Comparison of Band Combinations**

The use of reflectances calculated from previously measured spectra and sensor response functions only allow a comparison of location and width of bands as they relate to a particular feature such as vegetation. Other sensor characteristics such as noise, resolution, and calibration were not considered. Since a complete sensitivity analysis cannot be made without including the other sensor characteristics, the dynamic range of greenness was used as a measure of vegetation discrimination. It was assumed that the band combination that yielded the greatest dynamic range of greenness would be the best combination to discriminate vegetation from the soil background.

Greenness values calculated for four satellite sensors are shown in Figure 3 as a function of relative vegetation amount. The sensors and bands used are identified in the figure. Other than the Thematic Mapper band 6 (thermal-IR) and the thermal-IR bands of the AVHRR, all available bands for each sensor were used, although not all combinations are reported. Figure 3 shows the relation between greenness and the relative amount of green vegetation for the four sensors. The 6-D greenness (all bands other than the thermal-IR) of the Thematic Mapper had the greatest dynamic range. The Landsat-4 MSS (4-D GN) and the SPOT HRV (3-D GN) had nearly the same range. The 2-D GN of the NOAA-7 AVHRR had the smallest greenness range of the four sensors.
For a given sensor, different band combinations yield different values of greenness. This is shown in Figure 4 for the Landsat-4 MSS. The 4-D GN had the greatest dynamic range followed by the 2-D GN using band 2 (red) and band 4 (near-IR). These two lines provide a comparison of the PVI of Richardson and Wiegand (1977) and the 4-D GN of Kauth and Thomas (1976). The 2-D factor is about 90% of the 4-D GN. The 3-D GN using bands 1, 2, and 3 has a much smaller slope than does the 2-D GN using bands 2 and 4. The 2-D GN range for bands 1 and 3 is less than half that of the 4-D GN range.

Since the abscissa of Figures 3 and 4 extends from 0 to 1, the slopes of the greenness versus relative vegetation amount are a measure of the dynamic range. The slope, called the dynamic range of greenness (DRG), was calculated for a number of band combinations for the five sensors using linear regression. The results are given in Table 2 in descending order of the dynamic range for each sensor and for band combinations within sensors.

The 6-D GN for the Thematic Mapper proved to be the index with the greatest dynamic range (DRG = 40.6). However, two 5-D and one 4-D indices had the same value (the band combinations were ordered before the DRG values were rounded). For the TM, bands 4 and 7 appear to contribute most to vegetation discrimination. The DRG for the 3-D GN for bands 4, 5 and 7 is 98% of that for the 6-D GN, and for 3, 4, and 7 it is only slightly less than 98%. For the 2-D GN using 4 and 7 the DRG is almost 95% of the 6-D value. When band 7 was not included in the indices, the DRG was lower than for other combinations. However, the 2-D combination of bands 5 and 7 gave very low DRG values. Without either 5 or 7 (using only 1, 2, 3 and 4) the DRG is only 81% of the 6-D value. Without band 4 (near-IR), even a 5-D GN has a very low dynamic range of greenness.

The DRG for the 3-D GN of SPOT and the 4-D GN of Landsat-4 are 77% and 75% of the 6-D GN of TM, respectively. The 2-D NOAA DRG values are 60% as large as those of the TM. TM bands 1, 2,
TABLE 2 Dynamic Range of Greenness (DRG) for Various Band Combinations of Five Satellite Sensors

<table>
<thead>
<tr>
<th>Band Combination</th>
<th>DRG</th>
<th>Band Combination</th>
<th>DRG</th>
</tr>
</thead>
<tbody>
<tr>
<td>THEMATIC Mapper</td>
<td></td>
<td>SPOT HRV</td>
<td></td>
</tr>
<tr>
<td>1,2,3,4,5,7</td>
<td>40.6</td>
<td>1,2,3</td>
<td>31.4</td>
</tr>
<tr>
<td>2,3,4,5,7</td>
<td>40.6</td>
<td>2,3</td>
<td>29.4</td>
</tr>
<tr>
<td>1,3,4,5,7</td>
<td>40.6</td>
<td>Pan,3</td>
<td>25.4</td>
</tr>
<tr>
<td>3,4,5,7</td>
<td>40.6</td>
<td>1,3</td>
<td>21.1</td>
</tr>
<tr>
<td>1,2,4,5,7</td>
<td>40.0</td>
<td>1,2</td>
<td>2.6</td>
</tr>
<tr>
<td>4,5,7</td>
<td>39.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,2,3,4,7</td>
<td>39.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3,4,7</td>
<td>39.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4,7</td>
<td>38.4</td>
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<td></td>
</tr>
<tr>
<td>1,2,3,4,5</td>
<td>36.9</td>
<td>1,2,3,4</td>
<td>30.4</td>
</tr>
<tr>
<td>2,3,4,5</td>
<td>36.7</td>
<td>2,3,4</td>
<td>27.8</td>
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<td>1,3</td>
<td>20.0</td>
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<td>1,4</td>
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<td>1.2</td>
<td>2.4</td>
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<td>5,7</td>
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<td>1,2,3</td>
<td>3.0</td>
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<td>3,5</td>
<td>0.9</td>
<td></td>
<td></td>
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<td>NOAA-7</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1,2</td>
<td>24.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOAA-6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3, and 4, SPOT bands 1, 2, and 3, and Landsat-4 bands 1, 2, 3, and 4 have nearly the same DRG. For 2-D indices, TM 3 and 4, SPOT 2 and 3, and Landsat 2 and 4 have quite similar DRG Values, with the Landsat value being slightly less than the other two.

Concluding Remarks

When written in terms of simple summations, the calculation of n-space indices is rather straightforward. Computationally, a BASIC computer program of less than one page is sufficient to calculate the coefficients. Input requirements are one more data point (in n bands) than the number of indices required. For example, if only brightness and greenness are desired in 6-D from Thematic Mapper data, two soil points and one vegetation point are necessary. The selection of the soils data points is critical. On the other hand, the vegetation point is not critical. To investigate this, a vegetation point from each of the 12 wheat plots (which differed considerably in the amount of vegetation) was used to calculate greenness coefficients, with essentially no difference in the final result.

The soil line in n-space will usually not pass through the origin. As a consequence, the greenness will not necessarily be zero for bare soil (see Figures 3 and 4). When greenness values are to be compared among sensors, different band combinations, or different soil backgrounds, a more accurate measure would be to calculate the greenness for bare soil and subtract this value from all greenness val-
ues for the particular sensor, band combination, and soil background.

One can deduce from Figure 1 that if the slope of the soil line changes, the orthogonal distance to the green point will change. Thus, greenness in addition to brightness may be dependent on the reflectance properties of soils. The soil line developed from satellite imagery would be an average of many soils and atmospheric path radiance conditions, and may not always be the most appropriate. Soil lines developed for groups of soils having similar reflectance properties may improve the estimation of vegetation amounts using satellite data.

Indices formed by linear combinations of reflectances provide a means of comparing different sensors and identifying those band combinations that are most sensitive to the feature of interest. Reducing satellite data to usable form requires costly computer time. Data such as those in Table 2 may prove useful for selecting the minimum number of bands needed to obtain a desired degree of vegetation discrimination.

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Appendix

The following is a numerical example of the calculation of $n$-space coefficients. Reflectance data for four data points representative of wet and dry soil, green vegetation, and senesced vegetation (chosen to represent a yellow point) for four bands are given in Table 3. With these data it is possible to calculate up to three indices in four bands ($m = 3$, $n = 4$). A fifth data point would be necessary to calculate the fourth index.

Using reflectance data from Table 3 in eq. (1) gives

\[
b_1 = 15.10 - 7.59 = 7.51, \\
b_2 = 20.32 - 11.79 = 8.55, \\
b_3 = 28.73 - 15.52 = 13.21, \\
b_4 = 32.45 - 17.65 = 14.80. \\
\]

The normalization factor [Eq. (2)] is

\[
B = (7.51^2 + 8.53^2 + 13.21^2 \\
+ 14.80^2)^{1/2} = 22.86.
\]

The coefficients for brightness [Eq. (3)]

| TABLE 3 Spectral Reflectance Data (%) for the Example Calculation of 4-Space Coefficients |
| % REFLECTANCE MSS IN LANDSAT BANDS |
| 1  | 2  | 3  | 4  |
| Dry soil  | 15.10 | 20.32 | 28.73 | 32.45 |
| Wet soil   | 7.59  | 11.79 | 15.52 | 17.65 |
| Green vegetation | 3.45  | 2.80  | 28.51 | 43.82 |
| Senesced vegetation | 11.58 | 17.59 | 25.71 | 31.36 |
SPECTRAL INDICES IN $N$-SPACE

are

$$A_{1,1} = \frac{7.51}{22.86} = 0.3285,$$
$$A_{1,2} = \frac{8.53}{22.86} = 0.3731,$$
$$A_{1,3} = \frac{13.21}{22.86} = 0.5779,$$
$$A_{1,4} = \frac{14.80}{22.86} = 0.6473.$$

These coefficients are used in Eq. (4) to obtain brightness.

From Table 3, the dry soil (any soil point could be used) and the green vegetation point differences are

$$\left( X_g - X_s \right)_1 = 3.45 - 15.10 = -11.65,$$
$$\left( X_g - X_s \right)_2 = 2.80 - 20.32 = -17.52,$$
$$\left( X_g - X_s \right)_3 = 28.51 - 28.73 = -0.22,$$
$$\left( X_g - X_s \right)_4 = 43.82 - 33.45 = 11.37.$$

Equation (6) is evaluated by summing the products of the above differences with the brightness coefficients, i.e.,

$$D_{2,1} = (-11.65)(0.3285) + (-17.52)(0.3731) + (-0.22)(0.5779) + (11.37)(0.6473) = -3.131.$$

Using this value in Eq. (5) for each band yields

$$g_1 = -11.65 - (-3.131)(0.3285) = -10.62,$$
$$g_2 = -17.52 - (-3.131)(0.3731) = -16.35,$$
$$g_3 = -0.22 - (-3.131)(0.5779) = 1.60,$$
$$g_4 = 11.37 - (-3.131)(0.6473) = 13.40.$$

The normalization factor [Eq. (7)] is

$$G = \left[ \left( -10.62 \right)^2 + \left( -16.35 \right)^2 + 1.60 + (13.40)^2 \right]^{1/2} = 23.71.$$

The coefficients of greenness [Eq. (8)] are

$$A_{2,1} = \frac{-10.62}{23.71} = -0.4480,$$
$$A_{2,2} = \frac{-16.35}{23.71} = -0.6896,$$
$$A_{2,3} = \frac{1.60}{23.71} = 0.0670,$$
$$A_{2,4} = \frac{13.40}{23.71} = 0.5650.$$

The third step is to calculate the coefficients of yellowness. Again the differences between the yellowness point and the soil point are formed for each band, i.e.,

$$\left( X_y - X_s \right)_1 = 11.58 - 15.10 = -3.52,$$
$$\left( X_y - X_s \right)_2 = 17.59 - 20.32 = -2.73,$$
$$\left( X_y - X_s \right)_3 = 25.71 - 28.73 = -3.02,$$
$$\left( X_y - X_s \right)_4 = 31.36 - 32.45 = -1.09.$$

$D_{3,1}$ is calculated using Eq. (11) in the same manner as $D_{2,1}$. Its value is -4.625. Also $D_{3,2}$ is calculated using Eq. (12). Its value is 2.640. Then,

$$y_1 = -3.52 - \left[ \left( -4.625 \right)(0.3285) + (2.640)(-0.4480) \right] = -0.8179,$$
$$y_2 = -2.73 - \left[ \left( -4.625 \right)(0.3731) + (2.640)(0.6896) \right] = 0.8162.$$
\[ y_3 = -3.02 - \left[ (-4.625)(0.5779) + (2.640)(0.0670) \right] = -0.5241, \]
\[ y_4 = -1.09 - \left[ (-4.625)(0.6473) + (2.640)(0.5650) \right] = 0.4122. \]

The normalization factor [Eq. (13)] is 1.3341, and the coefficients of yellowness [Eq. (14)] are

\[ A_{3,1} = -0.8179/1.334 = -0.6130, \]
\[ A_{3,2} = 0.8162/1.334 = 0.6118, \]
\[ A_{3,3} = -0.5241/1.334 = -0.3928, \]
\[ A_{3,4} = 0.4122/1.334 = 0.3089. \]

To check for orthogonality use Eq. (21). With the brightness and greenness,

\[ (0.3285)(-0.4480) + (0.3731)(-0.6896) + (0.5779)(0.0670) + (0.6473)(0.5650) = 0.000014, \]

a number within rounding error of zero. Orthogonality checks between brightness and yellowness, and greenness and yellowness, should also be calculated.

References


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